A Microscopic Investigation of Force Generation in a Permanent Magnet Synchronous Machine

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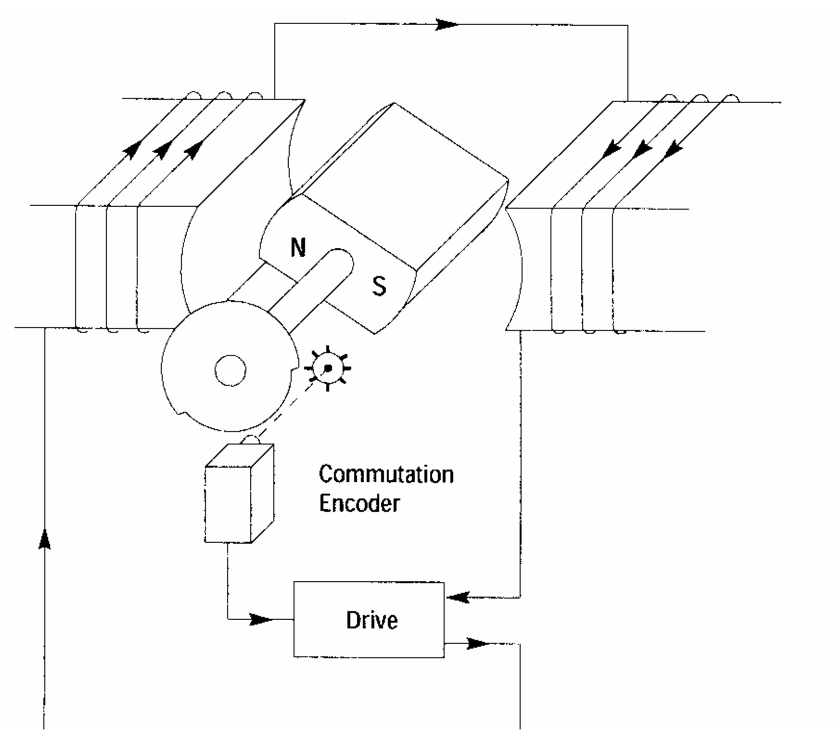
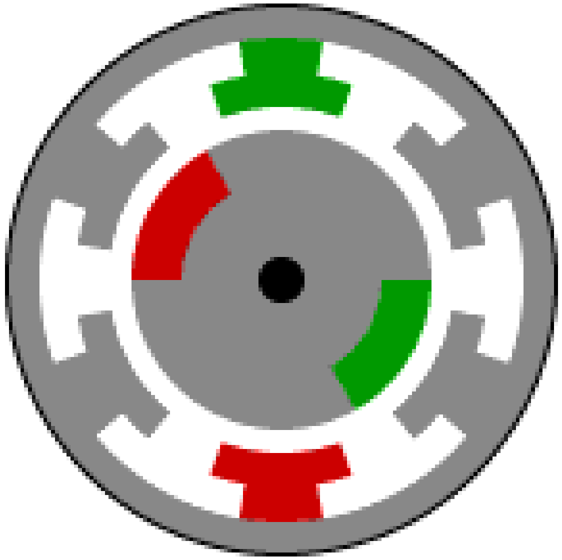
Outline

- Torque and force characteristics
  - Structure of permanent magnet (PM) synchronous machine
  - Define macroscopic view of torque
  - Define microscopic view of force
  - Microscopic torque and force characteristics of PM synchronous machines
- Tangential and radial force ripple minimization
  - Field reconstruction (FR) method
  - Optimization using FR method

Permanent Magnet Synchronous Machine

- Has stator windings in stator slots
- Uses permanent magnets as magnetic source on the rotor





Macroscopic Machine Model

$$\mathbf{v}_{abcs} = \mathbf{r}_s \mathbf{i}_{abcs} + \frac{d}{dt}\boldsymbol{\lambda}_{abcs}$$

$$\boldsymbol{\lambda}_{abcs} = \mathbf{L}_s \mathbf{i}_{abcs} + \boldsymbol{\lambda}'_{abcpm}$$

$$\boldsymbol{\lambda}'_{abcpm} = \begin{bmatrix} \lambda_{apm} \\ \lambda_{bpm} \\ \lambda_{cpm} \end{bmatrix} = \begin{bmatrix} \lambda'_m \sin(\theta_r) \\ \lambda'_m \sin(\theta_r - \frac{2\pi}{3}) \\ \lambda'_m \sin(\theta_r + \frac{2\pi}{3}) \end{bmatrix}$$

$$T_e = \frac{\partial W_c}{\partial \theta_r} = \frac{P}{2}\left(i_{as}\frac{\partial \lambda_{aspm}}{\partial \theta_r} + i_{bs}\frac{\partial \lambda_{bspm}}{\partial \theta_r} + i_{cs}\frac{\partial \lambda_{cspm}}{\partial \theta_r} + \frac{\partial W_{pm}}{\partial \theta_r}\right)$$

Machine Model in Rotor Frame of Reference

$$v_{qs}^r = r_s i_{qs}^r + \omega_r \lambda_{ds}^r + p\lambda_{qs}^r$$

$$v_{ds}^r = r_s i_{ds}^r - \omega_r \lambda_{qs}^r + p\lambda_{ds}^r$$

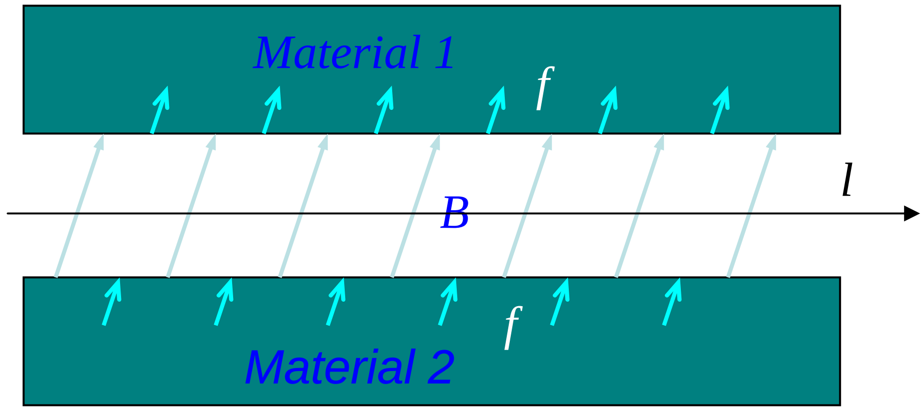
$$\lambda_{qs}^r = L_{ss} i_{qs}^r$$

$$\lambda_{ds}^r = L_{ss} i_{ds}^r + \lambda_m^{'}$$

$$T_e = \frac{3}{2}\frac{P}{2}\lambda^{'}{}_m i_{qs}^r + T_{cog}(\theta_r)$$

- Average torque not function of *d*-axis current

Field-Based Solution of Magnetic Forces



Maxwell Stress Tensor (Neglecting *z*-component of flux density)

Force Density:

$$f_t = B_n \cdot B_t / \mu_0$$

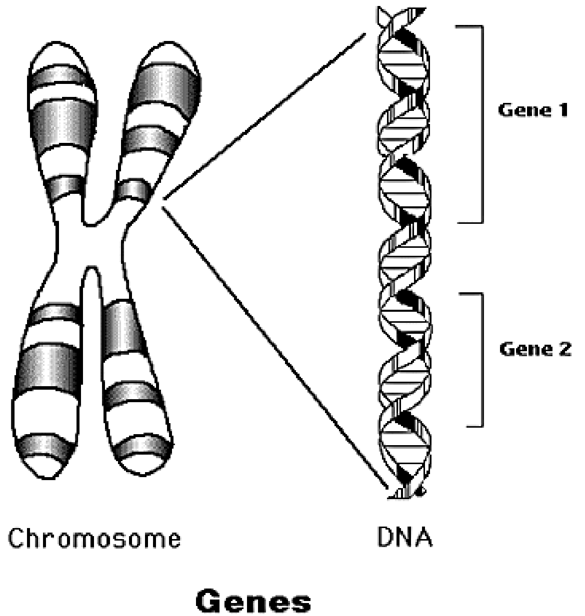
$$f_n = (B_n^2 - B_t^2) / 2\mu_0$$

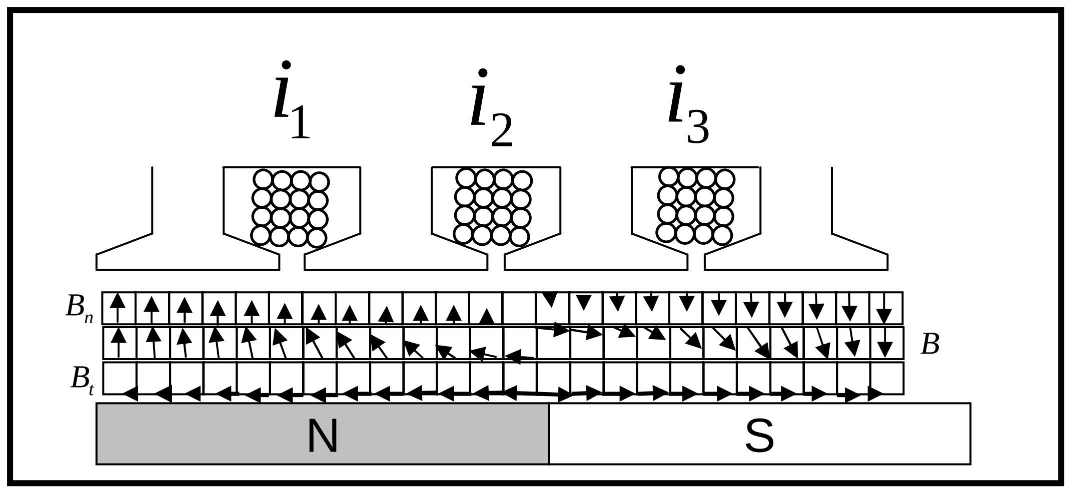
Overall Force:

$$F_t = \int f_t \cdot dl \qquad T_e = F_t \cdot L_{stack} \cdot R_{contour}$$

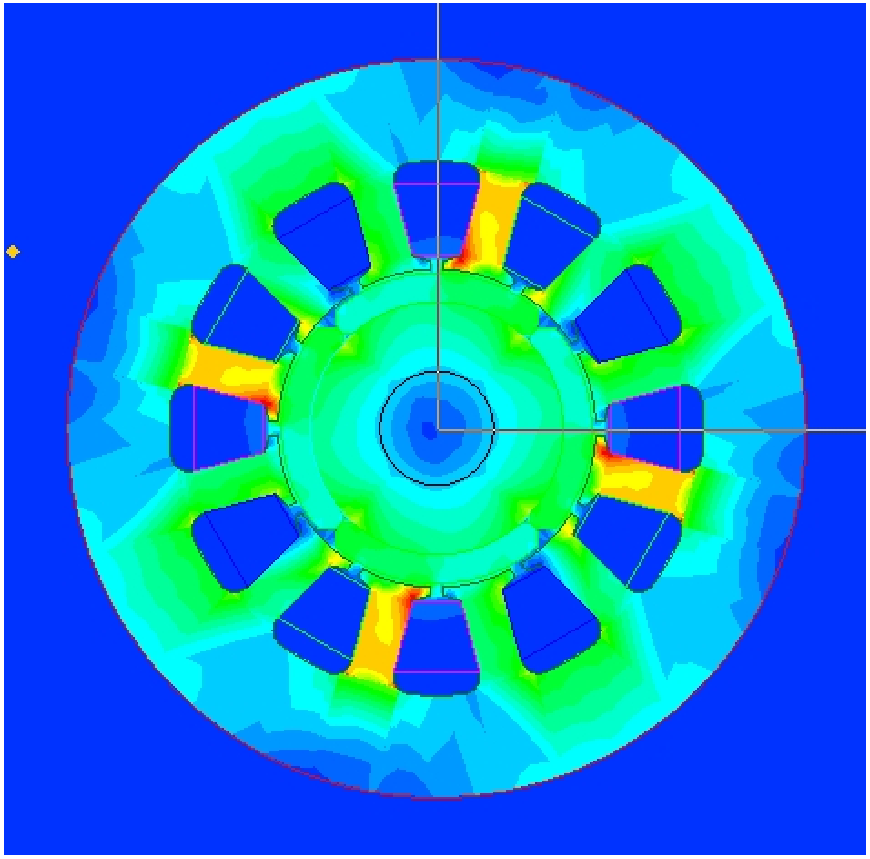
$$F_n = \int f_n \cdot dl$$

Microscopic View of Electric Machines



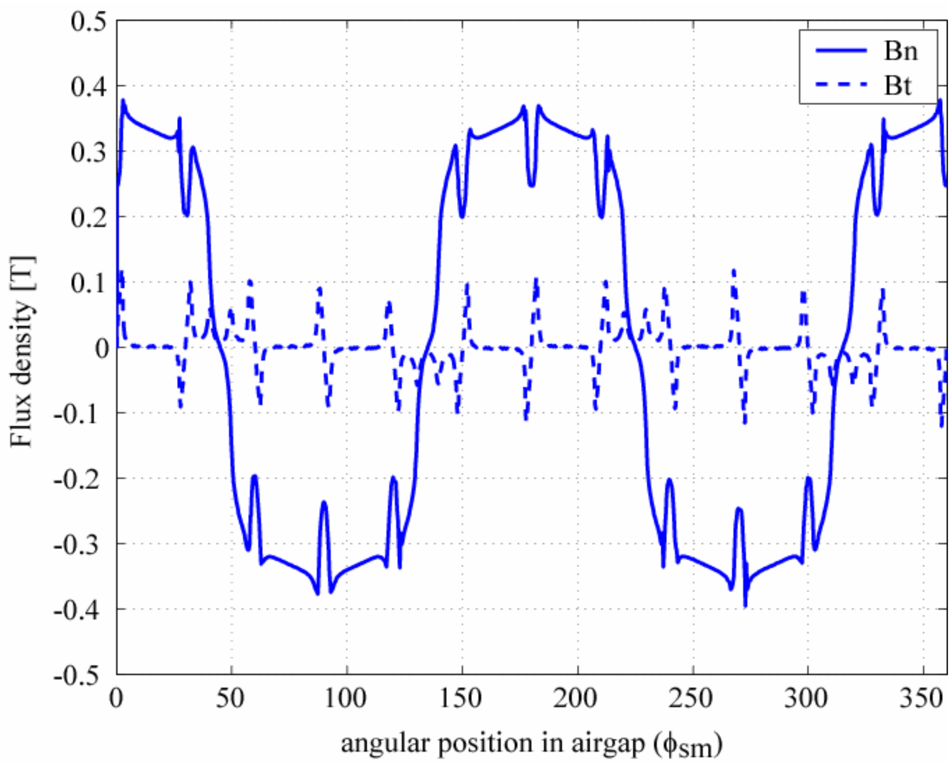


Cross-section of PM Machine Studied



3-phase, 4-pole, 12-slot, surface-mounted, 1 HP, 2000 rpm

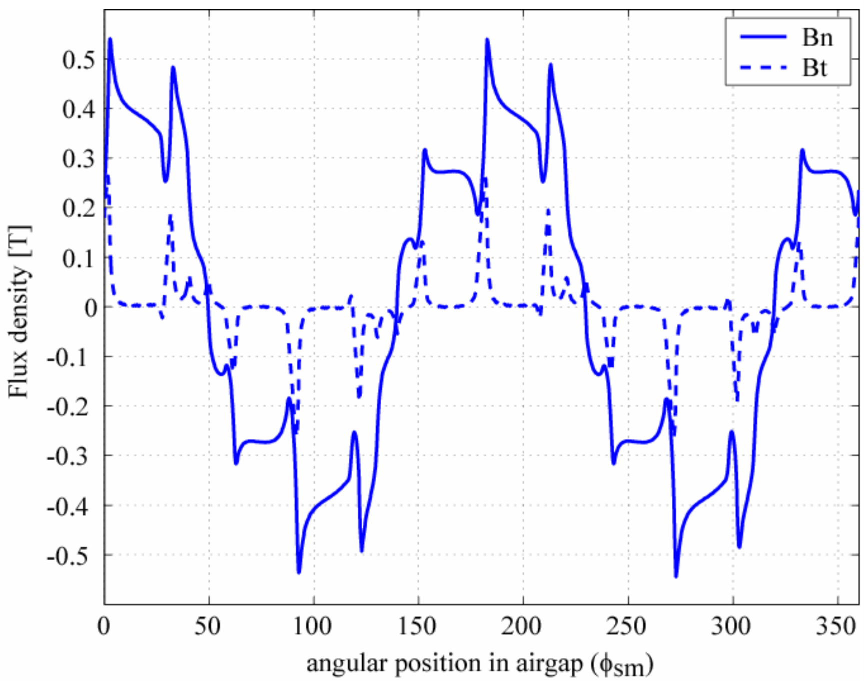
Flux and Force Density Distribution by PMs

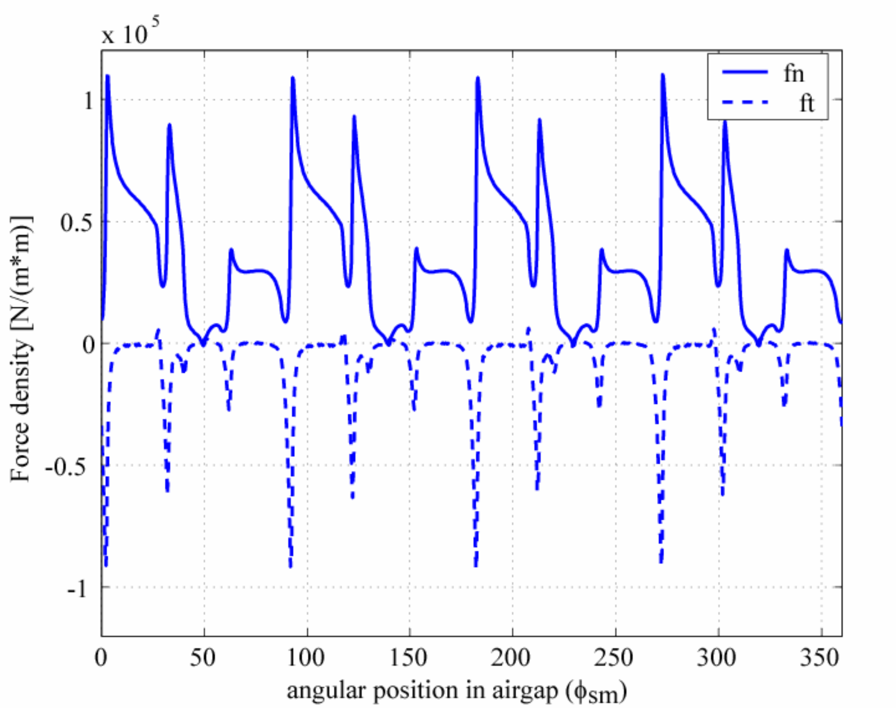


Distribution of B_t and B_n generated by permanent magnets

Average Forces: $F_t = 0, F_n = 5807$ N/m

Flux and Force Density Distribution – $i_{qs}^{r} = 4.6$ A



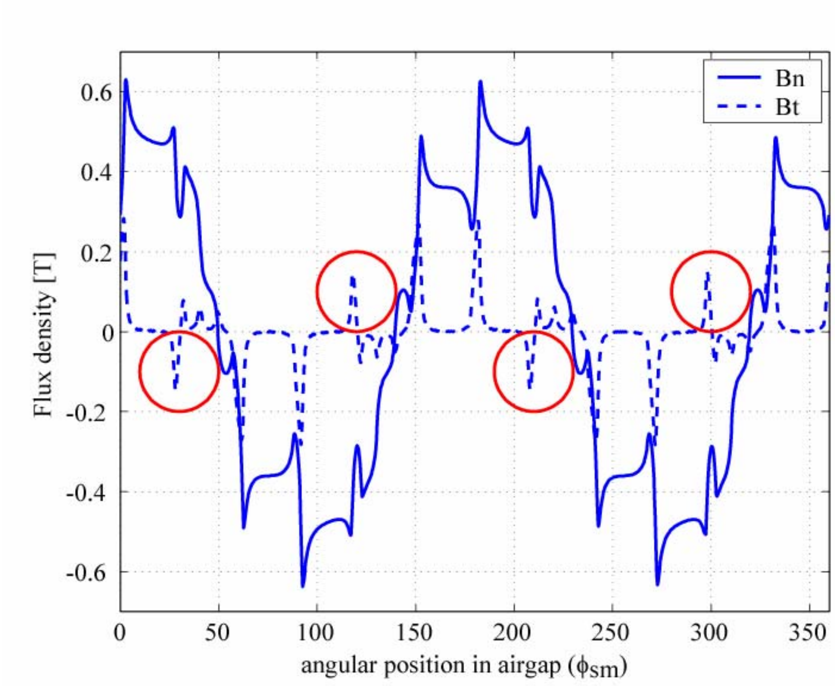


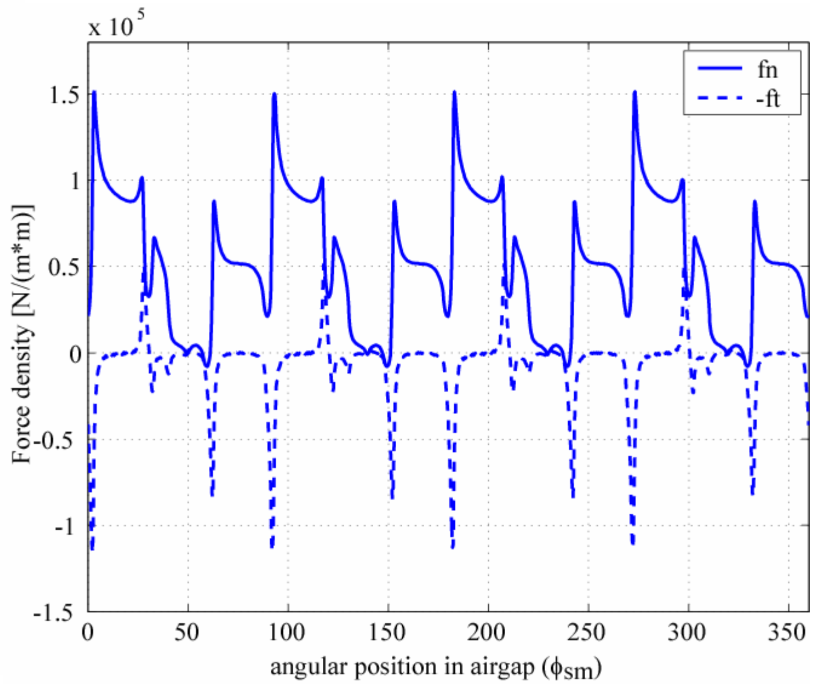
Distribution of B_t and B_n at single rotor position when $i_{ds}^{r} = 0, i_{qs}^{r} = 4.6$ A

Distribution of f_t and f_n at single rotor position when $i_{ds}^{r} = 0, i_{qs}^{r} = 4.6$ A

Average Forces: $F_t = 1219, F_n = 6239$ N/m

Flux and Force Density Distribution – $i_{ds}^{r} = 4.0$ A



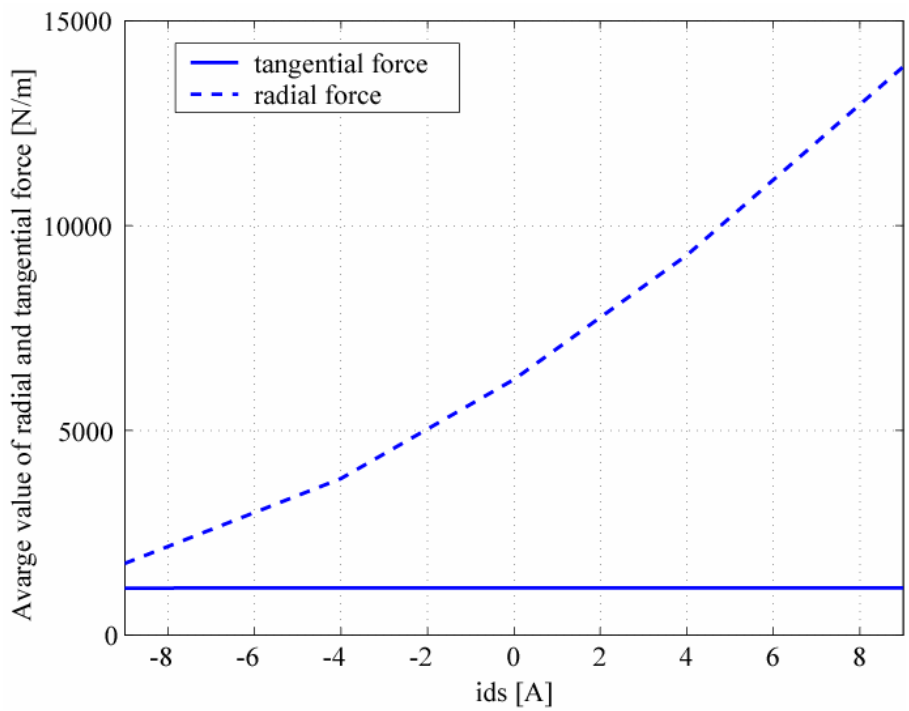


Distribution of B_t and B_n at single rotor position when $i_{ds}^{r} = 4.0, i_{qs}^{r} = 4.6$

Distribution of f_t and f_n at single rotor position when $i_{ds}^{r} = 4.0, i_{qs}^{r} = 4.6$

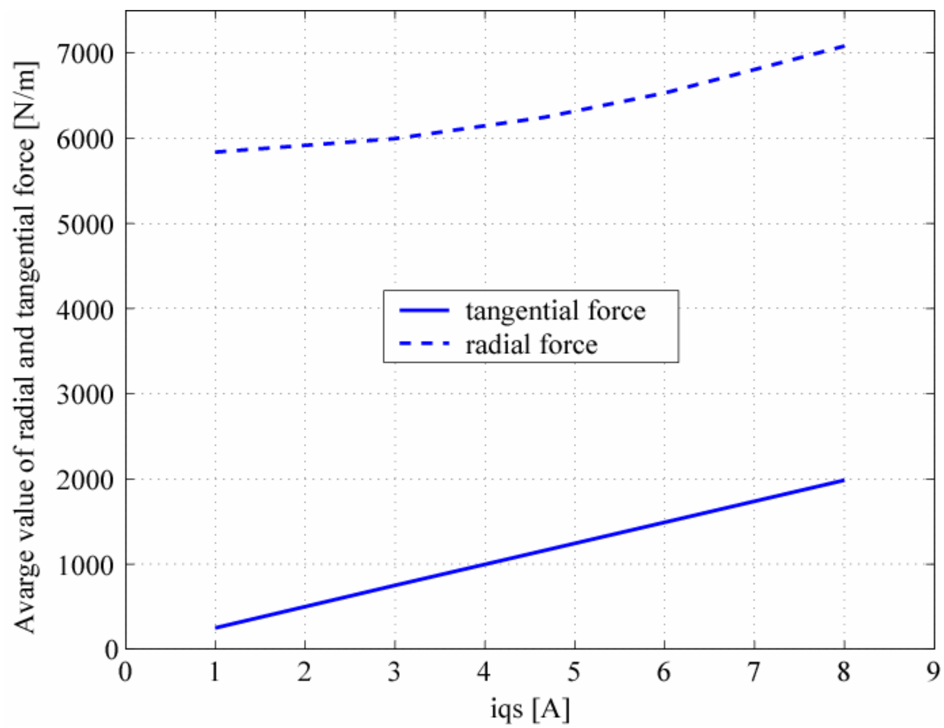
Average Forces: $F_t = 1218, F_n = 9132$ N/m

Effect of i_{ds}^{r} on Average Components of Force



d-axis current *(A)*	-9.0	-4.0	0	4.0	9.0
Average F_t *(N/m)*	1148	1154	1157	1160	1160
Average F_n *(N/m)*	1755	3852	6248	9278	13879

Effect of i_{qs}^{r} on Average Components of Force



q-axis current *(A)*	1	3	4.6	6	8
Average of Ft (N/m)	249	748	1157	1495	1985
Average of Fn (N/m)	5837	5996	6249	6531	7083

Detailed Force Density Expression

Source of Magnetic Field:

- Phase currents
- Permanent Magnets

If effect of saturation is neglected, then:

$$B_n = B_{npm} + B_{ns}$$

$$B_t = B_{tpm} + B_{ts}$$

$$f_t = \frac{1}{\mu_0}\left[B_{ts} + B_{tpm}\right]\left[B_{npm} + B_{ns}\right]$$

$$f_n = \frac{1}{2\mu_0}\left[(B_{npm} + B_{ns})^2 - (B_{ts} + B_{tpm})^2\right]$$

Components of Torque

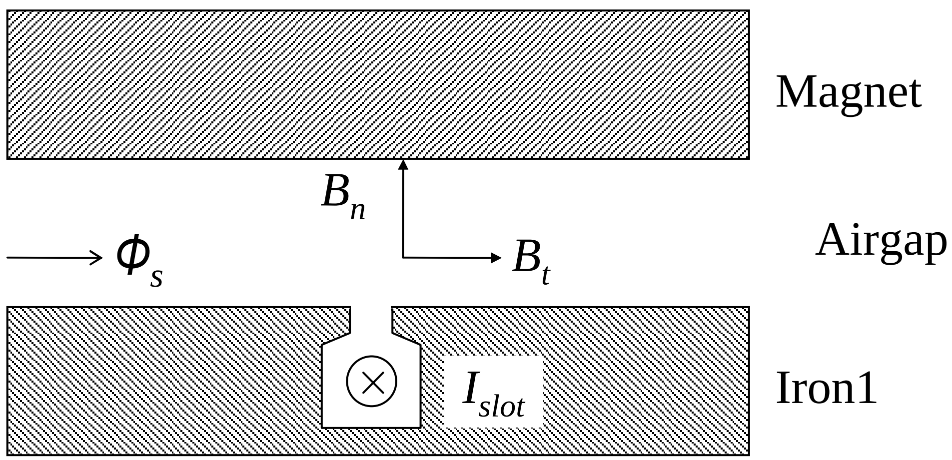
$B_{tpm} B_{npm}$ (zero average)

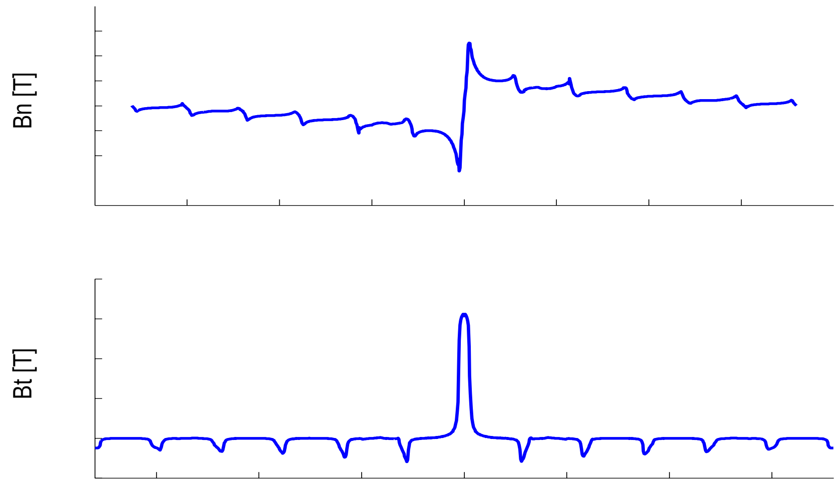
$B_{ns} B_{ts}$ (zero average)

$B_{ns} B_{tpm}$ (non-zero average)

$B_{ts} B_{npm}$ (non-zero average)

Flux Densities Generated by Current in Single Stator Slot





$$B_{tsk}(\phi_s) = I_{slot} \cdot f_1(\phi_s)$$

$$B_{nsk}(\phi_s) = I_{slot} \cdot f_2(\phi_s)$$

If set origin at center, then:

- f_1 is a even function.
- f_2 is a odd function.

Components of Tangential Force

$$B_{npm} = \sum_{k=1}^{\infty} B_{npmk} \cos(k\phi_r)$$

$$B_{tpm} = \sum_{k=1}^{\infty} B_{tpmk} \sin(k\phi_r)$$

$$B_{t\_as}(\phi_s) = i_{as} F_1(\phi_s)$$

$$B_{t\_bs}(\phi_s) = i_{bs} F_1(\phi_s - 120)$$

$$B_{t\_cs}(\phi_s) = i_{cs} F_1(\phi_s + 120)$$

$$F_1 = \sum_{k=1}^{\infty} F_{1k} \cos(k\phi_s)$$

$$B_{n\_as}(\phi_s) = i_{as} F_2(\phi_s)$$

$$B_{n\_bs}(\phi_s) = i_{bs} F_2(\phi_s - 120)$$

$$B_{n\_cs}(\phi_s) = i_{cs} F_2(\phi_s + 120)$$

$$F_2(\phi_s) = \sum_{k=1}^{\infty} F_{2k} \sin(k\phi_s)$$

Average Tangential Force Due to Fundamental Harmonics

$$\langle F_t \rangle = \frac{P}{2}\frac{3\pi}{\mu_0} \cdot (B_{npm1} \cdot F_{11} + B_{tpm1} \cdot F_{21}) \cdot 2\pi R \cdot I_{qs}^r$$

$$F_{21} = \frac{1}{\pi}\int_0^{2\pi} F_2(\phi_s) \cdot \sin(\phi_s) d\phi_s$$

$$F_{11} = \frac{1}{\pi}\int_0^{2\pi} F_1(\phi_s) \cdot \cos(\phi_s) d\phi_s$$

- *d*-axis current doesn't appear in average tangential force.

Components and Average Radial Force

$$f_n = \frac{1}{2\mu_0}\left[\begin{array}{l}(B_{npm} + B_{n-as} + B_{n-bs} + B_{n-cs})^2 - \\ (B_{t-as} + B_{t-bs} + B_{t-cs} + B_{tpm})^2\end{array}\right]$$

If consider only the fundamental components:

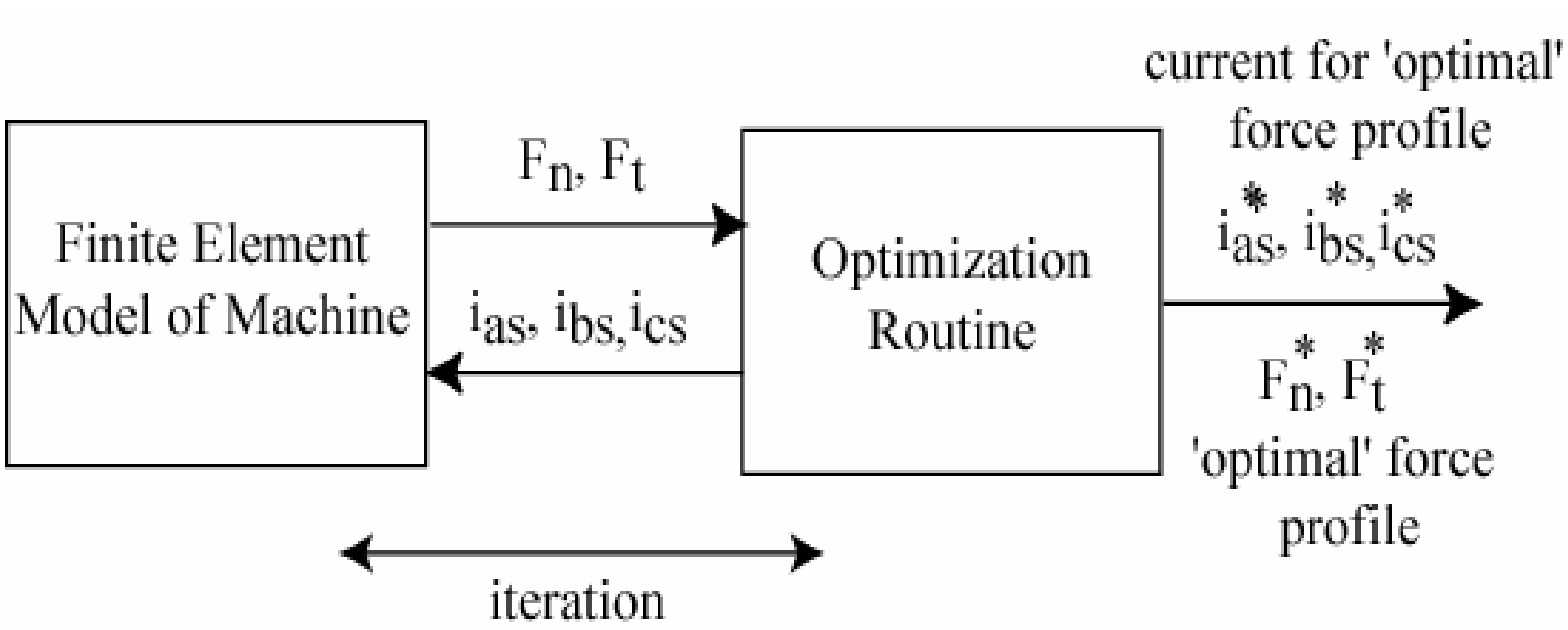
$$\langle F_n \rangle = \frac{P\pi R}{4\mu_0}[9(F_{21}{}^2 - F_{11}{}^2)(i_{qs}^{r\ 2} + i_{ds}^{r\ 2}) + B_{npm1}{}^2 - B_{tpm1}{}^2$$

$$+6(F_{21}B_{npm1} - F_{11}B_{tpm1})i_{ds}^r]$$

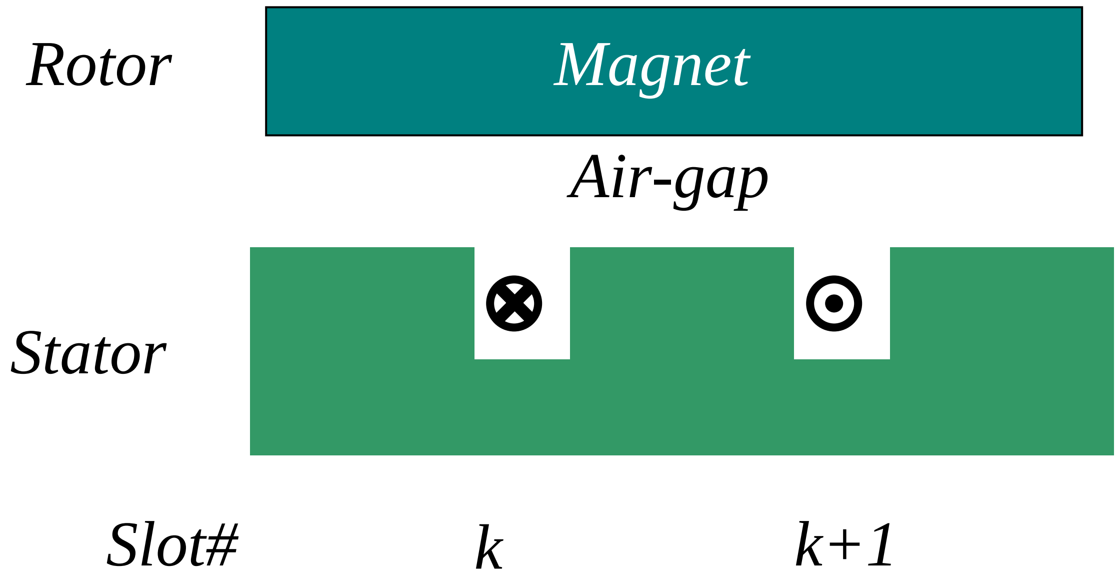
Problems in PM Synchronous Machine Applications

- Acoustic noise and vibration caused by:
  - Torque ripple
  - Harmonics in radial force
- Solutions for torque ripple mitigation:
  - Improve machine design to obtain better back-emf waveform
  - Employ excitation control methods to eliminate torque harmonics

Force Optimization using FEA



Field Reconstruction



$$B_{ns} = \sum_{k=1}^{L} B_{nsk}$$

$$B_{ts} = \sum_{k=1}^{L} B_{tsk}$$

$$B_n = B_{npm} + B_{ns}$$

$$B_t = B_{tpm} + B_{ts}$$

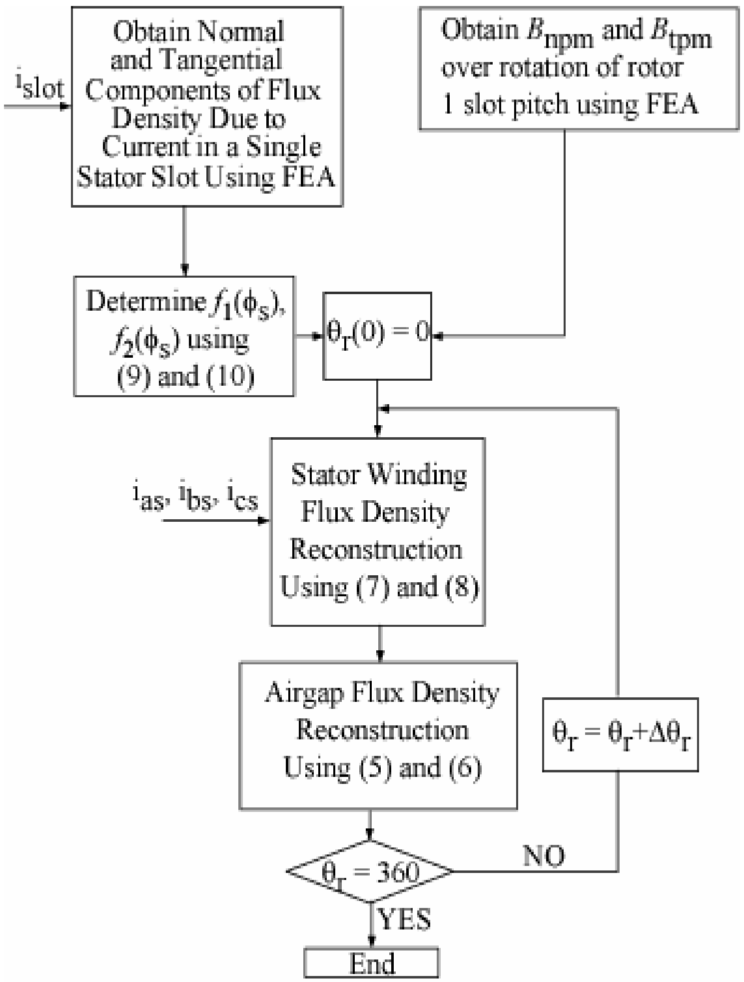
$$B_{tsk}(\phi_s) = I_{slot} \cdot f_1(\phi_s)$$

$$B_{nsk}(\phi_s) = I_{slot} \cdot f_2(\phi_s)$$

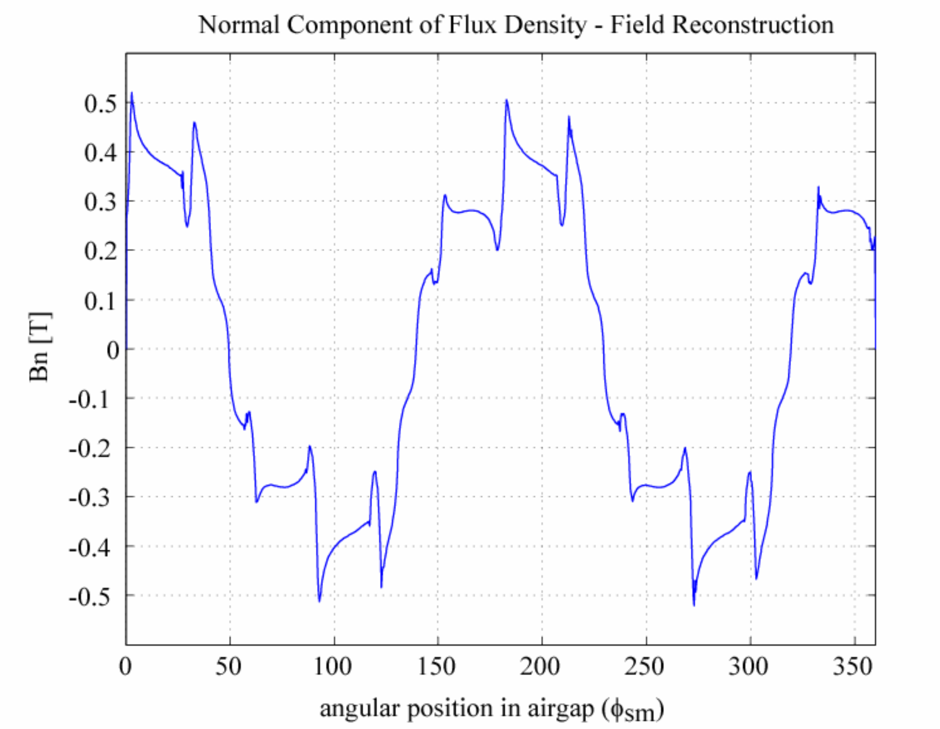
$$B_n = B_n(B_{nsk}, B_{npm})$$

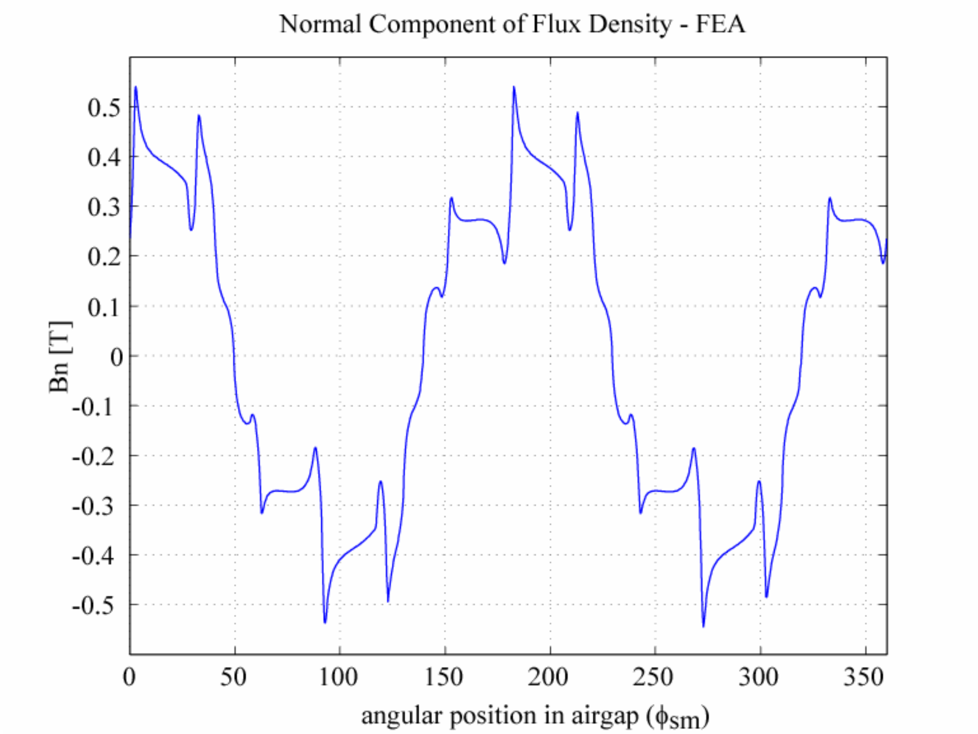
$$B_t = B_t(B_{tsk}, B_{tpm})$$

Field Reconstruction Method

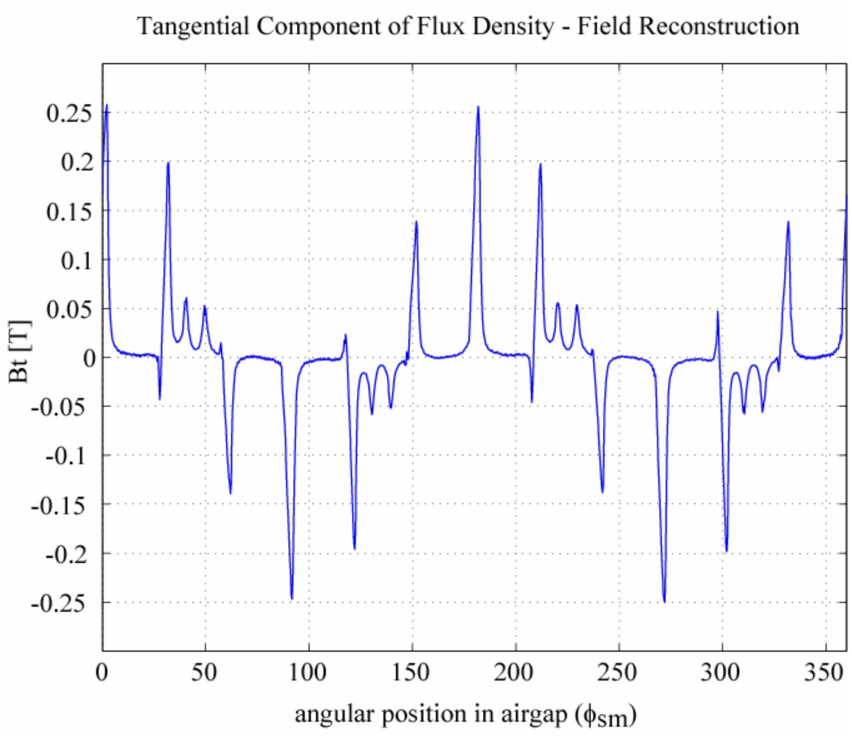


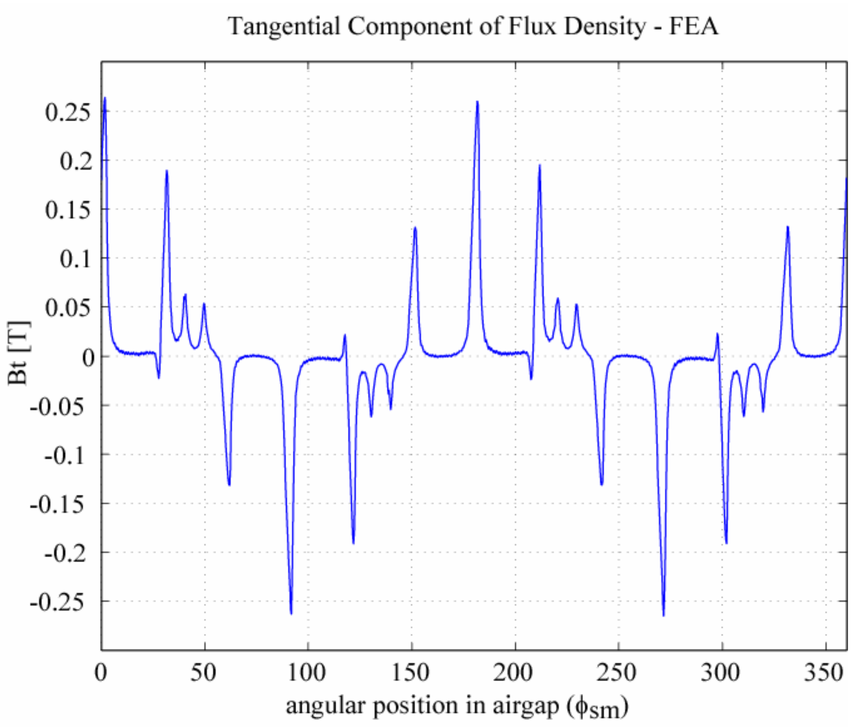
Field Reconstruction Results – Normal Component



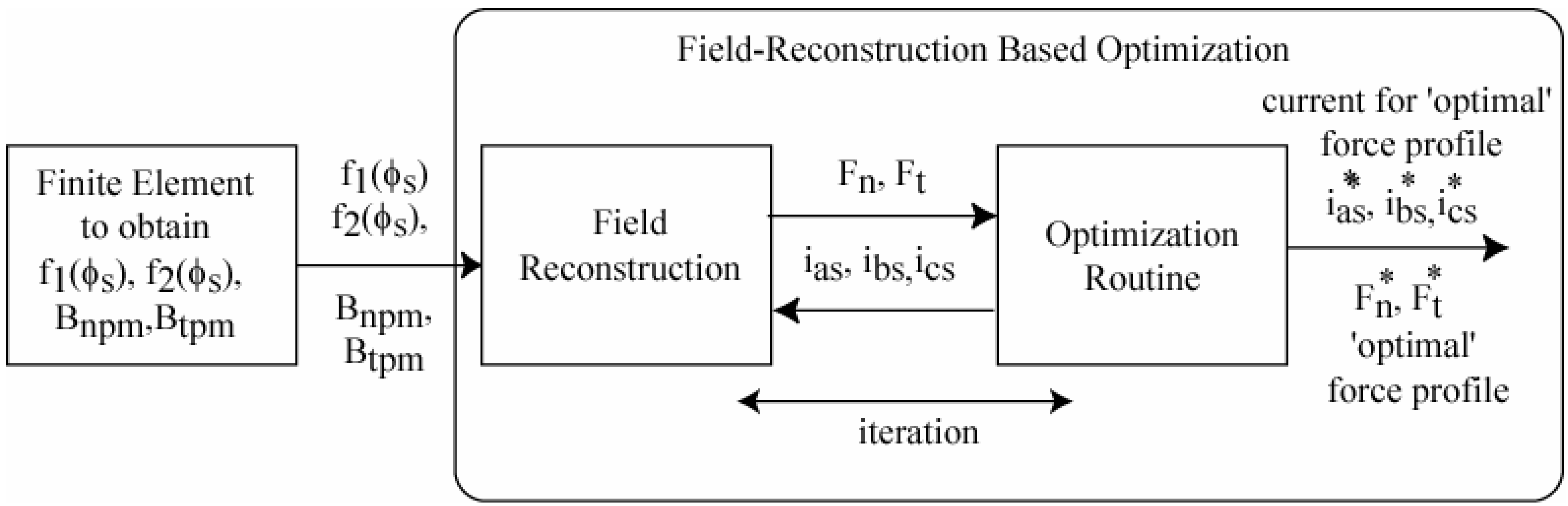


Field Reconstruction Results – Tangential Component





Force Optimization Using Field Reconstruction



Operation with Fixed Ft and Minimal Copper Loss

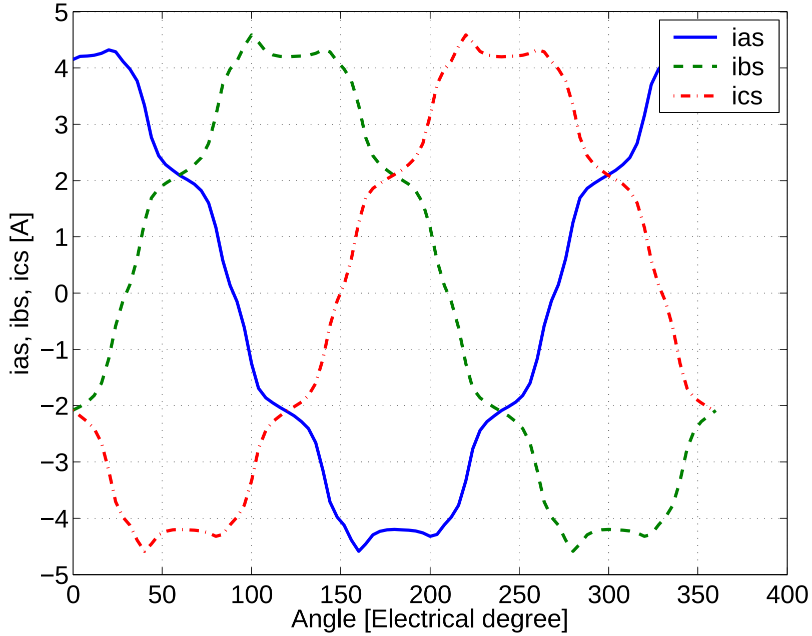
$$Min(i_{as}^2 + i_{bs}^2 + i_{cs}^2)$$

Subject to:

$$F_t = 1000 \text{ N/m}$$

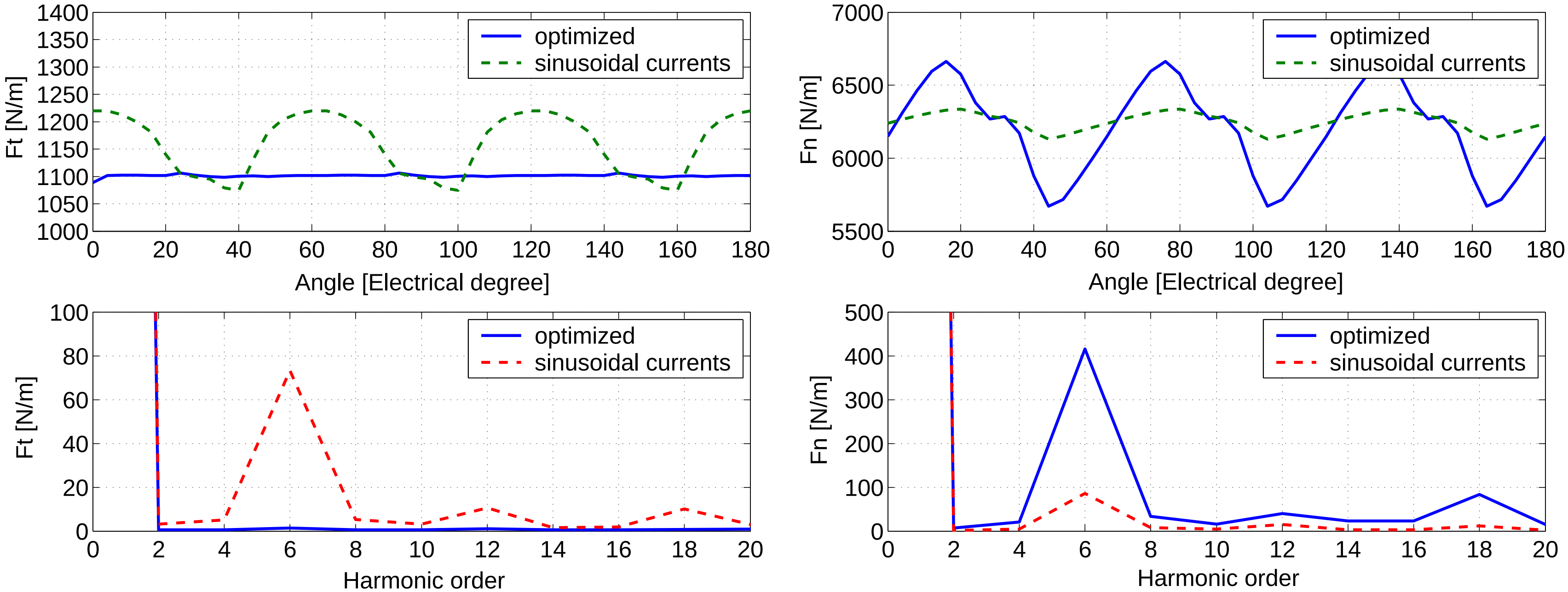
$$i_{as} + i_{bs} + i_{cs} = 0$$

$$-i_{\max} \le i_{as}, i_{bs}, i_{cs} \le i_{\max}$$



Phase current waveform

Waveform of Ft, Fn and Comparison with Sinusoidal Excitation



Peak-to-peak

	Ft (N/m)	Fn (N/m)
Optimal	7.3/1101	991/6198
Sinusoidal	145/1157	204/6248

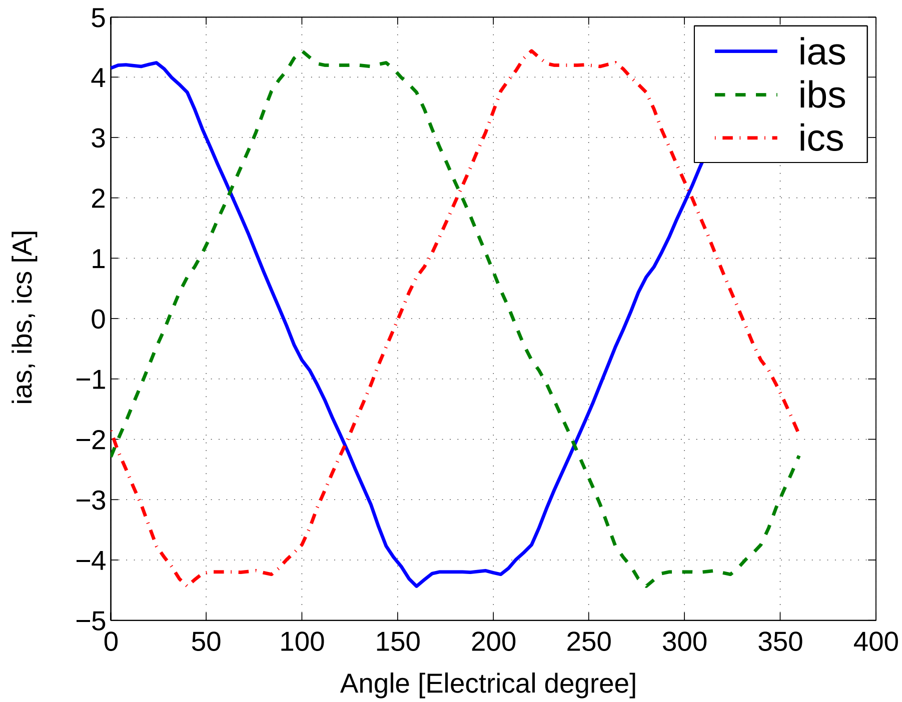
Operation with Fixed Fn and Ft

$$Min[(F_t - \langle F_t \rangle)^2$$
$$+(F_n - \langle F_n \rangle)^2]$$

Subject to:

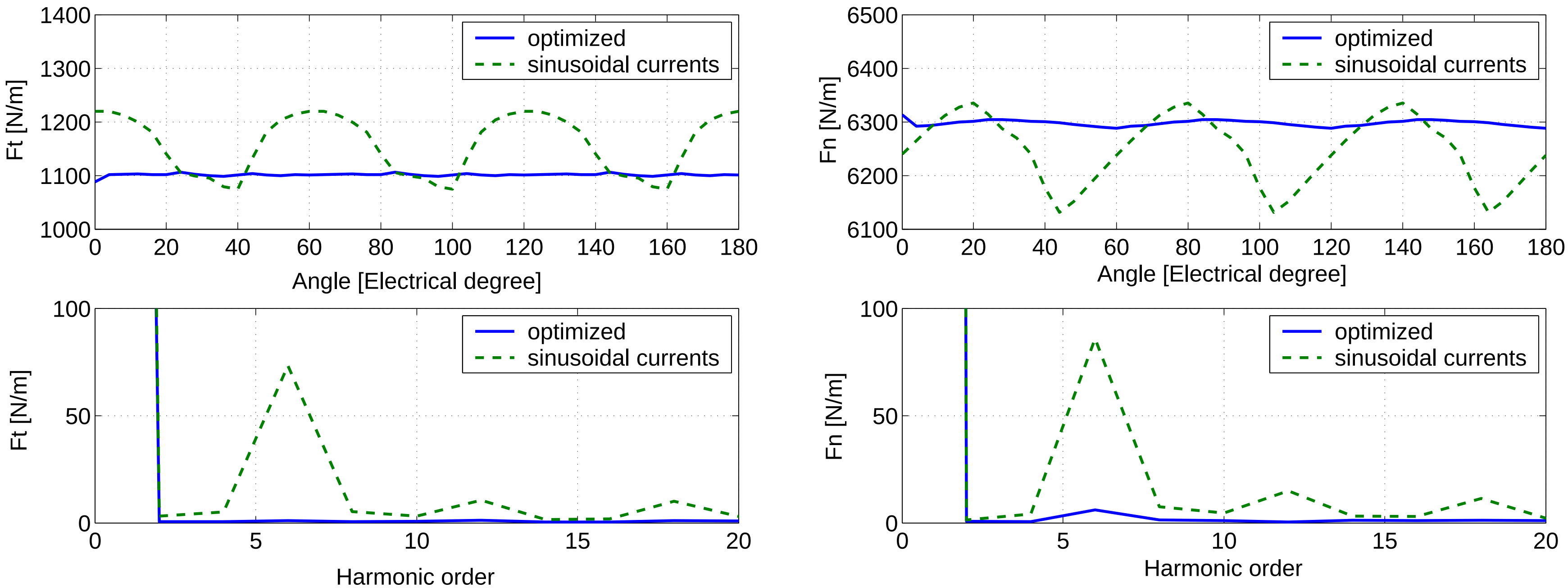
$$i_{as} + i_{bs} + i_{cs} = 0$$

$$-i_{\max} \le i_{as}, i_{bs}, i_{cs} \le i_{\max}$$



Phase current waveform

Waveform of Ft, Fn and Comparison with Sinusoidal Excitation



Peak-to-peak

	Ft (N/m)	Fn (N/m)
Optimal	7.3/1102	16/6297
Sinusoidal	145/1157	204/6248

Operation with Fixed Fn, Ft and Minimal Copper Loss

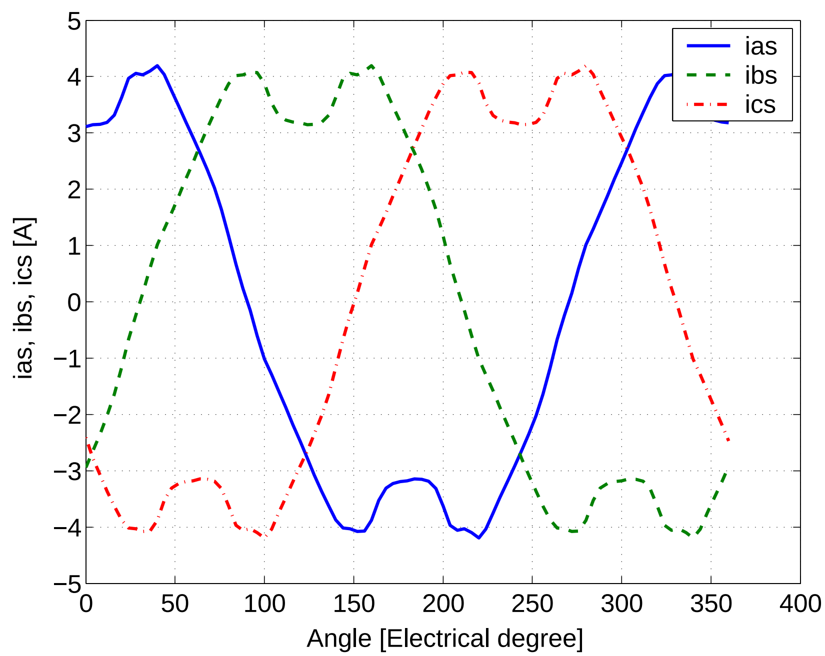
$$Min(i_{as}^2 + i_{bs}^2 + i_{cs}^2)$$

Subject to:

$$F_t = 1000 \quad \text{N/m}$$

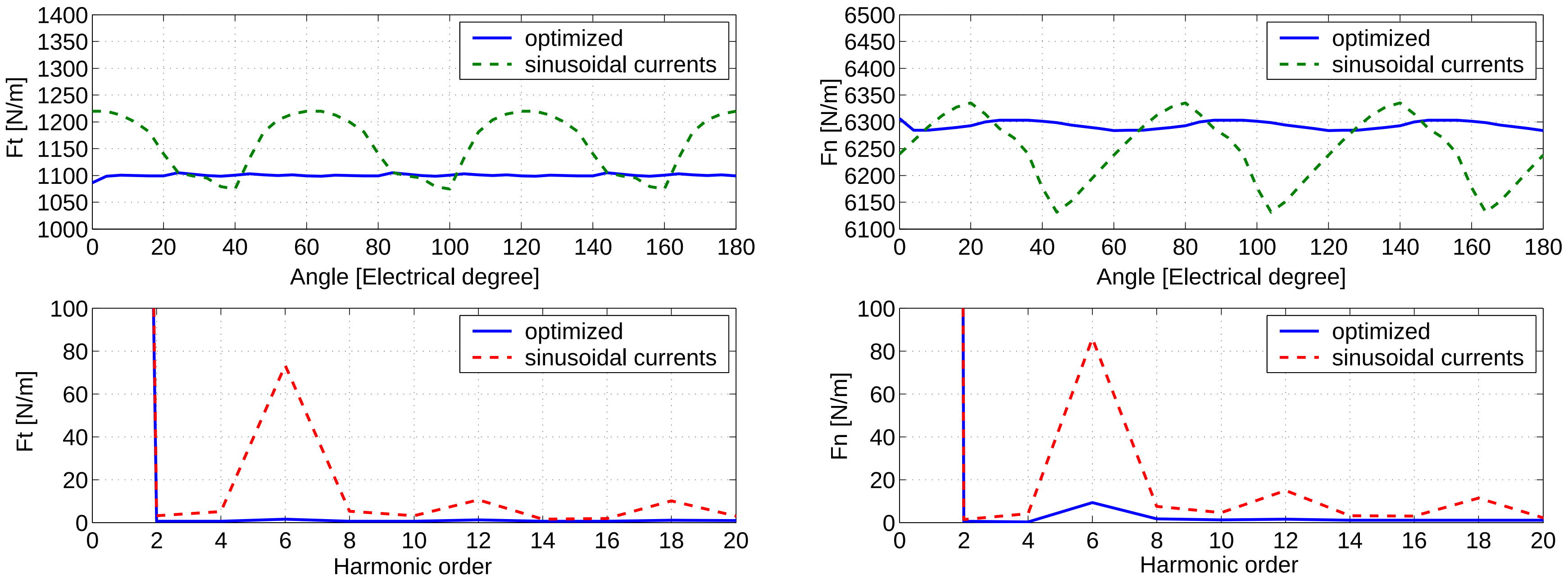
$$F_n = 6200 \quad \text{N/m}$$

$$-i_{\max} \le i_{as}, i_{bs}, i_{cs} \le i_{\max}$$



Phase current waveform

Waveform of Ft, Fn and Comparison with Sinusoidal Excitation



Peak-to-peak

	Ft (N/m)	Fn (N/m)
Optimal	6.6/1100	23/6293
Sinusoidal	145/1157	204/6248

Trapezoidal back-emf case: Fixed Fn, Ft and Minimal Copper Loss

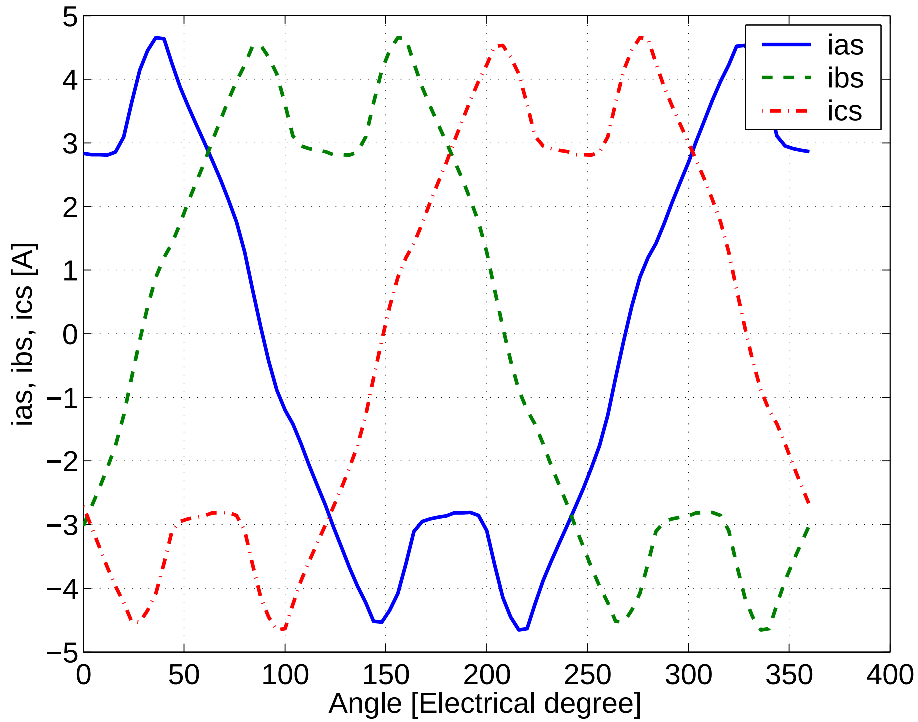
$$Min(i_{as}^2 + i_{bs}^2 + i_{cs}^2)$$

Subject to:

$$F_t = 1000$$

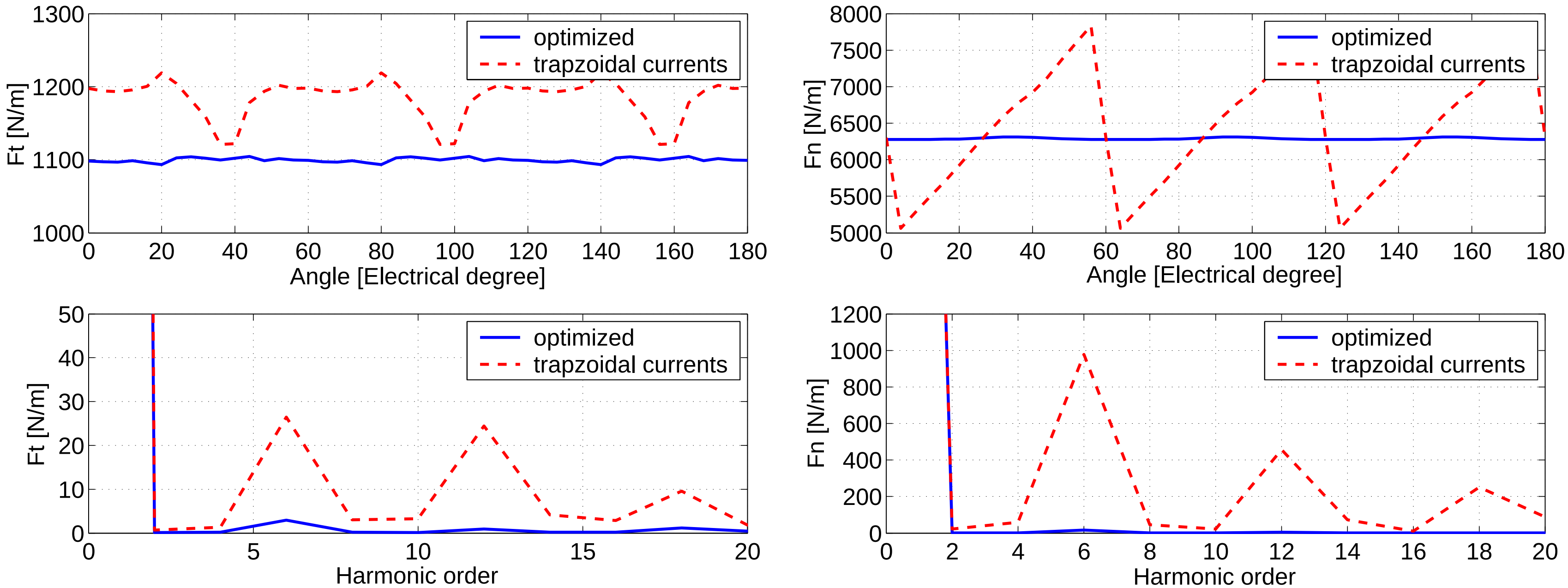
$$F_n = 6200$$

$$-i_{\max} \le i_{as}, i_{bs}, i_{cs} \le i_{\max}$$



Phase current waveform

Waveform of Ft, Fn and Comparison with Trapezoidal Excitation



Peak-to-peak

	Ft (N/m)	Fn (N/m)
Optimal	11/1100	31/6289
Trapezoidal	97/1185	2773/6427

Conclusions

- Microscopic investigation of forces leads to result that both d- and q-axis currents influence radial force (quadratic)
- Area of tangential force density relatively small – leading to larger radial than tangential force
- Opens the question - are alternative designs/excitation strategies possible to provide a more effective force profile?
- Field reconstruction is a time-efficient tool to consider alternative methods of excitation for control force profile