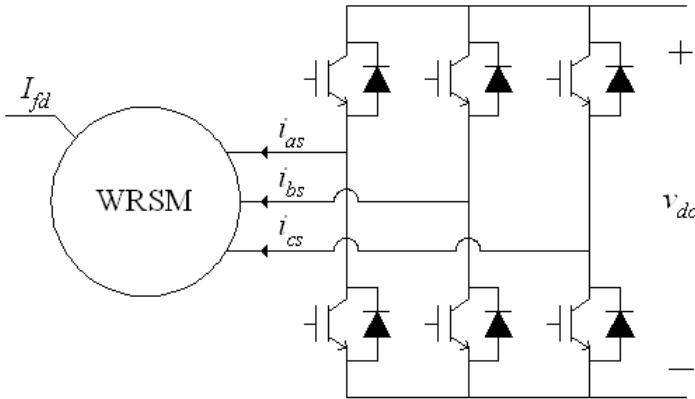


A Dynamic Magnetic Equivalent Circuit Model for the Design of Wound Rotor Synchronous Machines

Research of Xiaoqi (Ron) Wang
Advisor: Steve Pekarek

Department of Electrical and Computer Engineering
Purdue University

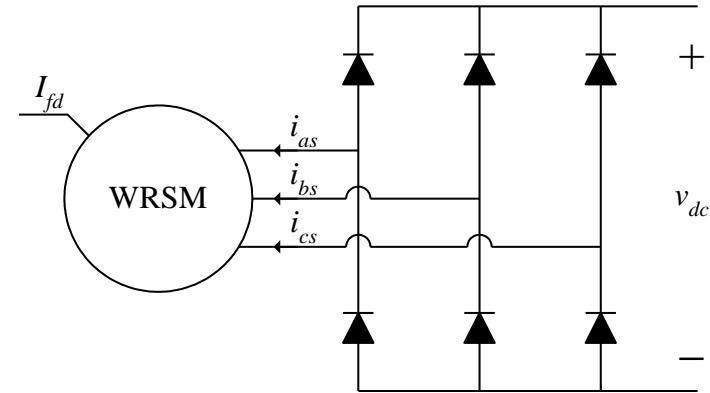
WRSM/Rectifier Design



WRSM/active rectifier

$$design = [d_{rc} \ l \ d_{rt} \ g \ d_{st} \ d_{bs} \ fw_{ss} \ fh_{rt} \ fw_{rt} \ \dots \ fw_{rp} \ N_s \ I_s \ \beta \ N_{fd} \ I_{fd} \ P_p \ ftipw \ ftiph]^T$$

$$I_{as} = I_s \cos(\theta_r + \beta)$$



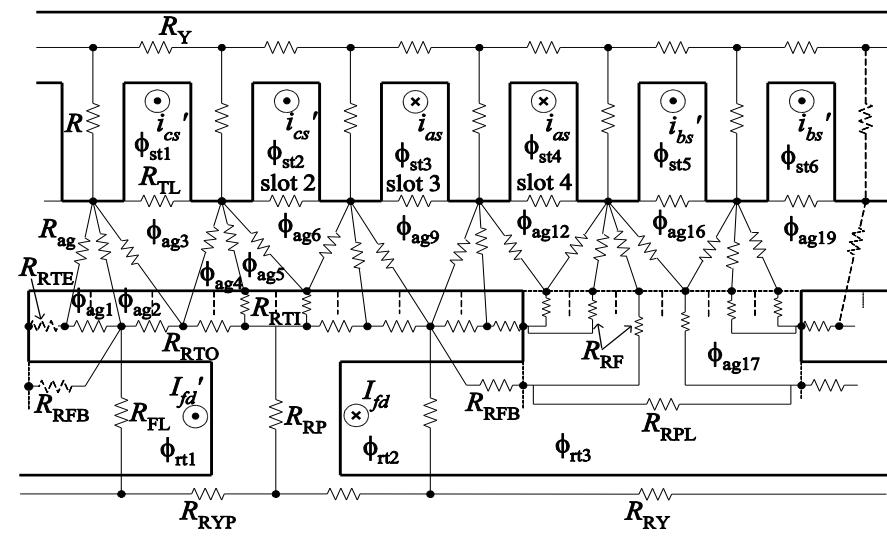
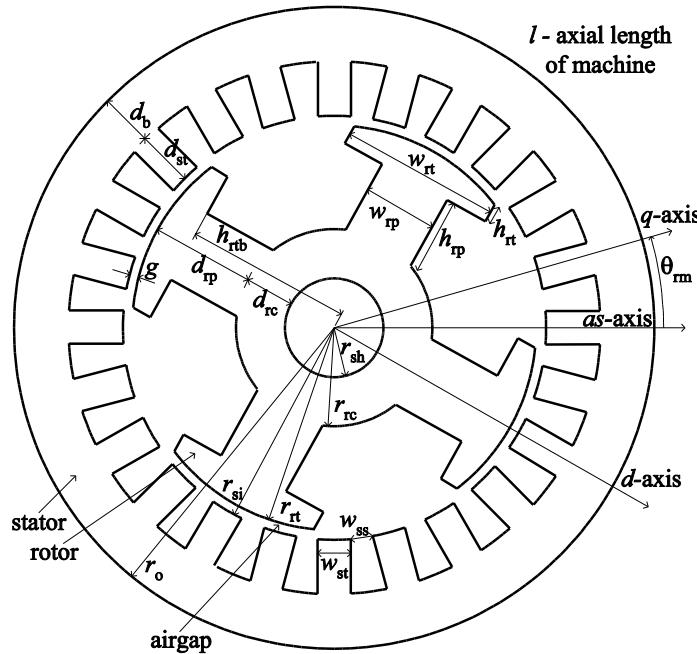
WRSM/pассив rectifier

$$design = [d_{rc} \ l \ d_{rt} \ g \ d_{st} \ d_{bs} \ fw_{ss} \ fh_{rt} \ fw_{rt} \ \dots \ fw_{rp} \ N_s \ N_{fd} \ I_{fd} \ P_p \ ftipw \ ftiph \ r_{dt} \ d_{num} \ d_{con}]^T$$

$$V_{dc} = f(I_{fd}, L_q, L_d)$$

MEC Model for Active Rectification

- A steady-state mesh-based MEC model for WRSMs



Performance Calculation

- Electromagnetic torque

$$T_e(\phi, \theta_r) = \left(\frac{P}{2}\right)^2 \sum_{j=1}^{na} \left(\frac{\phi_{aj}}{P_{aj}}\right)^2 \frac{\partial P_{aj}}{\partial \theta_r}$$

- Power loss

Resistive loss: $P_{res} = r_{fd} I_{fd}^2 + \frac{3}{2\pi} \int_0^{2\pi} r_s i_{as}^2(\theta_r) d\theta_r$

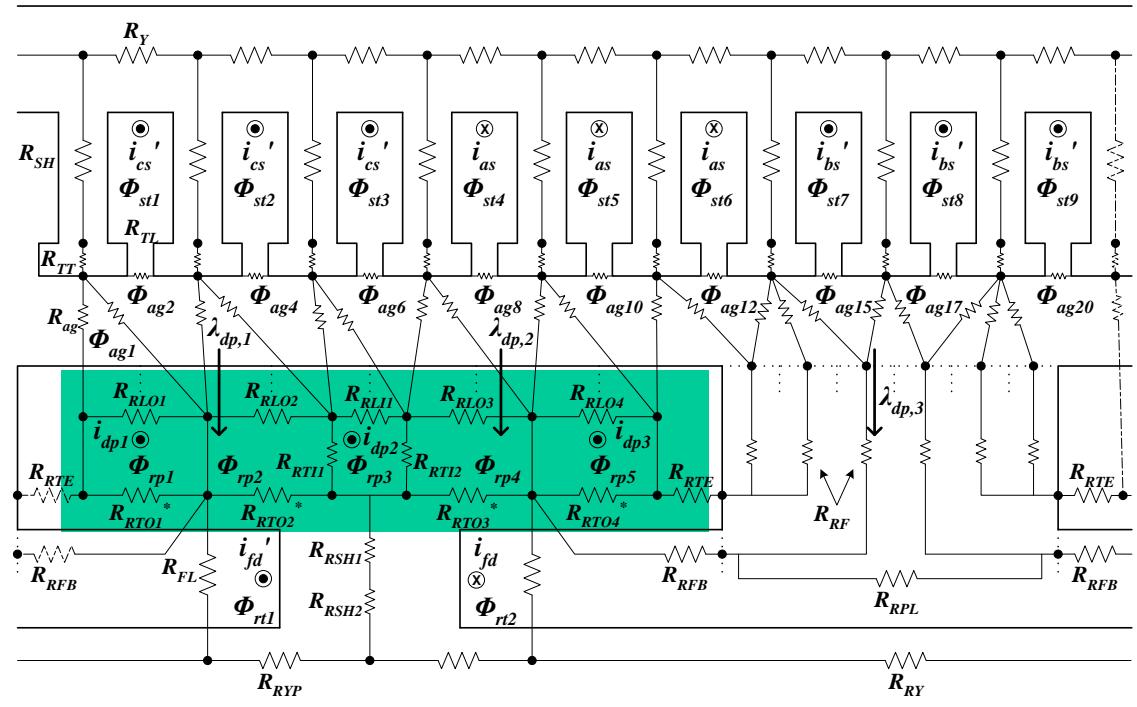
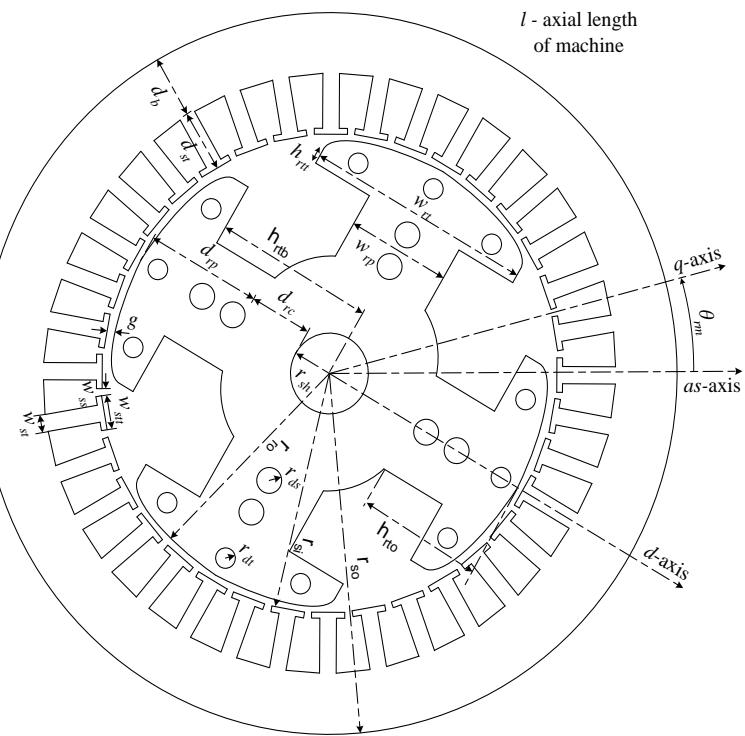
Core loss: $P_{core} = P_{ld,T} V_{ST} + P_{ld,Y} V_{SY}$

where $P_{ld}(B) = \underbrace{k_h f_{eq}^{\alpha-1} \left(\frac{B_{max}}{B_b} \right)^\beta f}_{Hysteresis\ Loss} + \underbrace{\frac{k_e f}{B_b^2} \int_0^T \left(\frac{dB}{dt} \right)^2 dt}_{Eddy\ Current\ Loss}$ and $f_{eq} = \frac{2}{(B_{max} - B_{min})^2 \pi^2} \int_0^T \left(\frac{dB}{dt} \right)^2 dt$

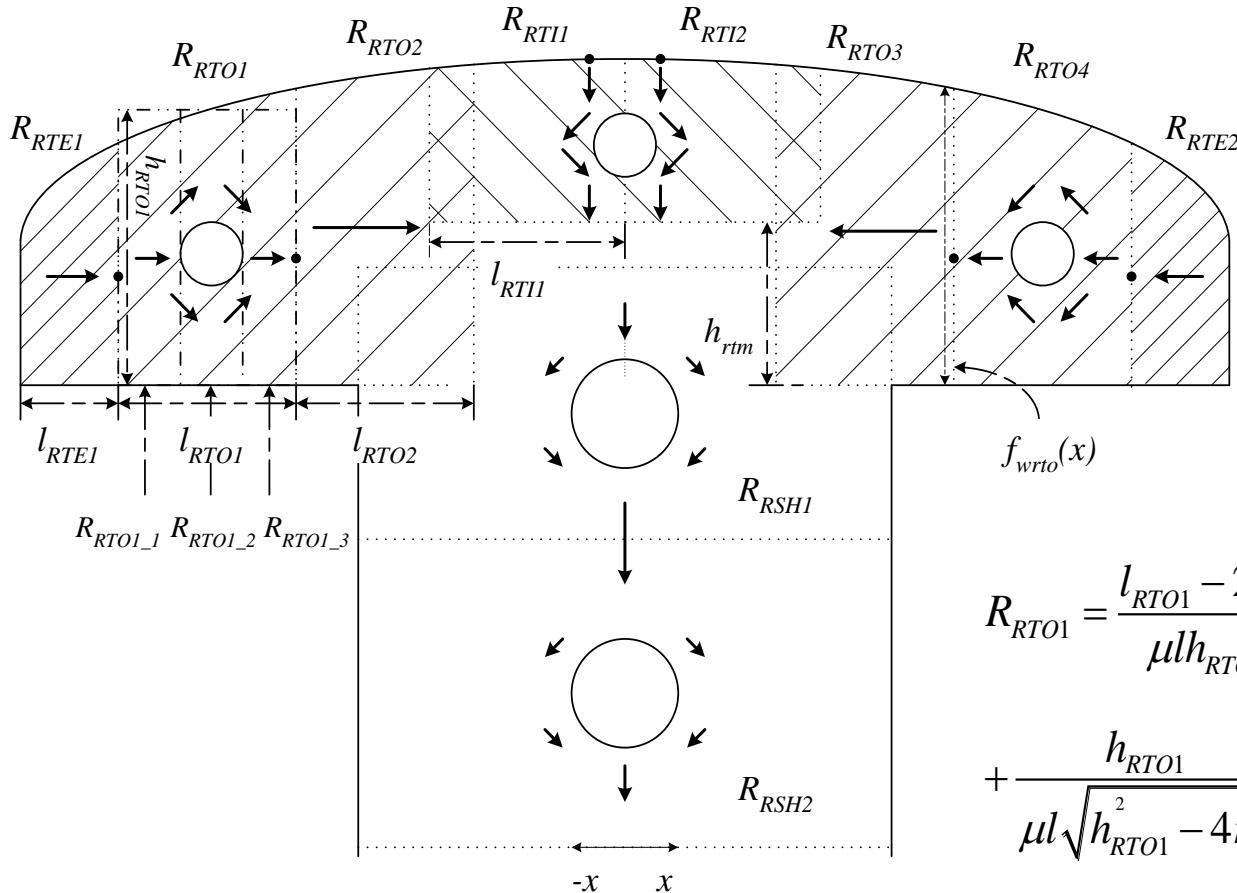
Conduction loss: $P_{cond} = 3V_{drop} \frac{1}{2\pi} \int_0^{2\pi} |i_{as}(\theta_r)| d\theta_r$

Total loss: $P_{loss} = P_{res} + P_{core} + P_{cond}$

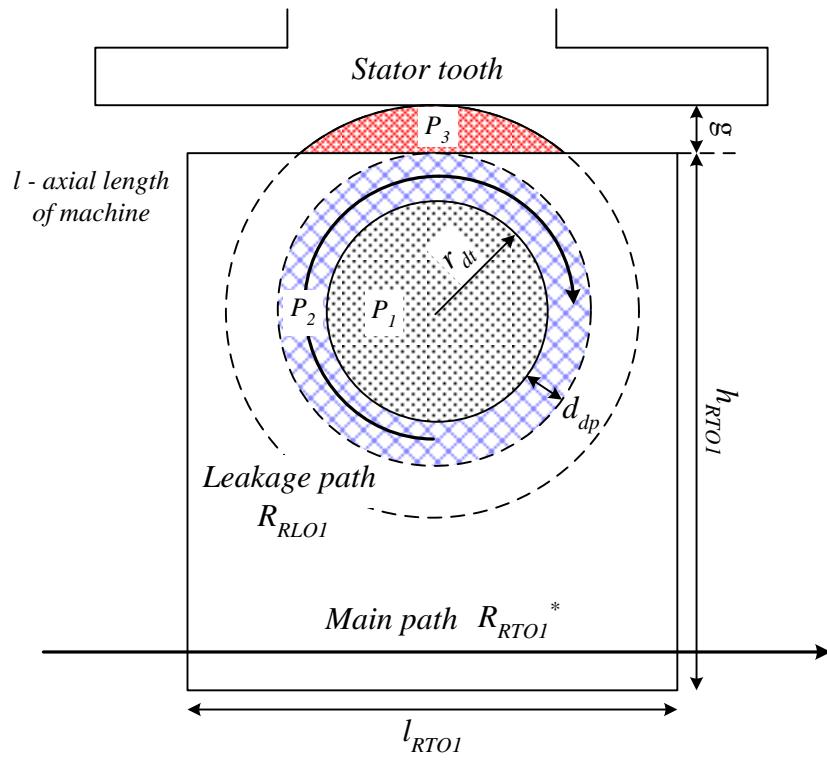
Dynamic MEC Network for Passive Rectification



Rotor Pole Flux Tubes



Rotor Pole Tip Leakage Flux Tubes



- P_1 : leakage inside the bar.
- P_2 : leakage in the steel.
- P_3 : leakage in the airgap.

$$d_{dp} = \alpha_{dp} (h_{RTO1} - 2r_{dt})$$

$$\frac{1}{R_{RLO1}} = \frac{\mu_0 l}{8\pi} + \underbrace{\frac{\mu l}{2\pi} \ln(\frac{d_{dp} + r_{dt}}{r_{dt}})}_{P_2}$$

$$+ \underbrace{\frac{\mu_0 l}{2} \ln(\frac{\sqrt{2g(d_{dp} + r_{dt}) + g^2} + g + d_{dp} + r_{dt}}{d_{dp} + r_{dt}})}_{P_3}$$

$$\frac{1}{R_{RTO1}^*} = \frac{1}{R_{RTO1}} - \frac{1}{R_{RLO1}}$$

Damper Bar Placement

- Odd number damper bars: one of the bars is located in the center of the most inner two R_{RTI} sections.
- Even number damper bars: there is no hole in the center of the most inner two R_{RTI_i} sections, but they are symmetrically distributed on the two sides of the rest of the rotor pole sections.
- Arbitrary number and radius of damper bars can be applied:

$$\mathbf{damper_rtip} = [\dots \quad r_{dt3} \quad r_{dt2} \quad r_{dt1} \quad r_{dt2} \quad r_{dt3} \quad \dots]$$

- Arbitrary vertical depth of the damper bars can be assigned by adjusting the scaling factor α_{dp} .

Steady-State KVL MEC Model

System equation: $\mathbf{A}_R^{(nl \times nl)} \boldsymbol{\Phi}_1^{(nl \times 1)} = \mathbf{F}_1^{(nl \times 1)}$

where $\boldsymbol{\Phi}_1 = [\phi_{st1} \ \dots \ \phi_{stns} \ \phi_{rt1} \ \dots \ \phi_{rtnr} \ \phi_{ag1} \ \dots \ \phi_{agns} \ \phi_{rp1} \ \dots \ \phi_{rpn}]^T$

and $\mathbf{F}_1 = [\mathbf{F}_{st}^{(ns \times 1)^T} \ \mathbf{F}_{rt}^{(nr \times 1)^T} \ \mathbf{0}^{(na \times 1)^T} \ \mathbf{F}_{rp}^{(np \times 1)^T}]^T$

Stator and rotor MMF source: $\mathbf{F}_{st}^{(ns \times 1)} = \mathbf{N}_{abc}^{(ns \times 3)} \mathbf{i}_{abcs}^{(3 \times 1)}, \mathbf{F}_{rt}^{(nr \times 1)} = \mathbf{N}_{rt}^{(nr \times 1)} I_{fd} = [-1 \ 1 \ 0]^T N_{fd} I_{fd}$

Damper winding MMF source: $\mathbf{F}_{rp}^{(np \times 1)}(j) = \mathbf{N}_{dp}^{(np \times nd)}(j, k) \mathbf{i}_{dp}^{(nd \times 1)}(k)$

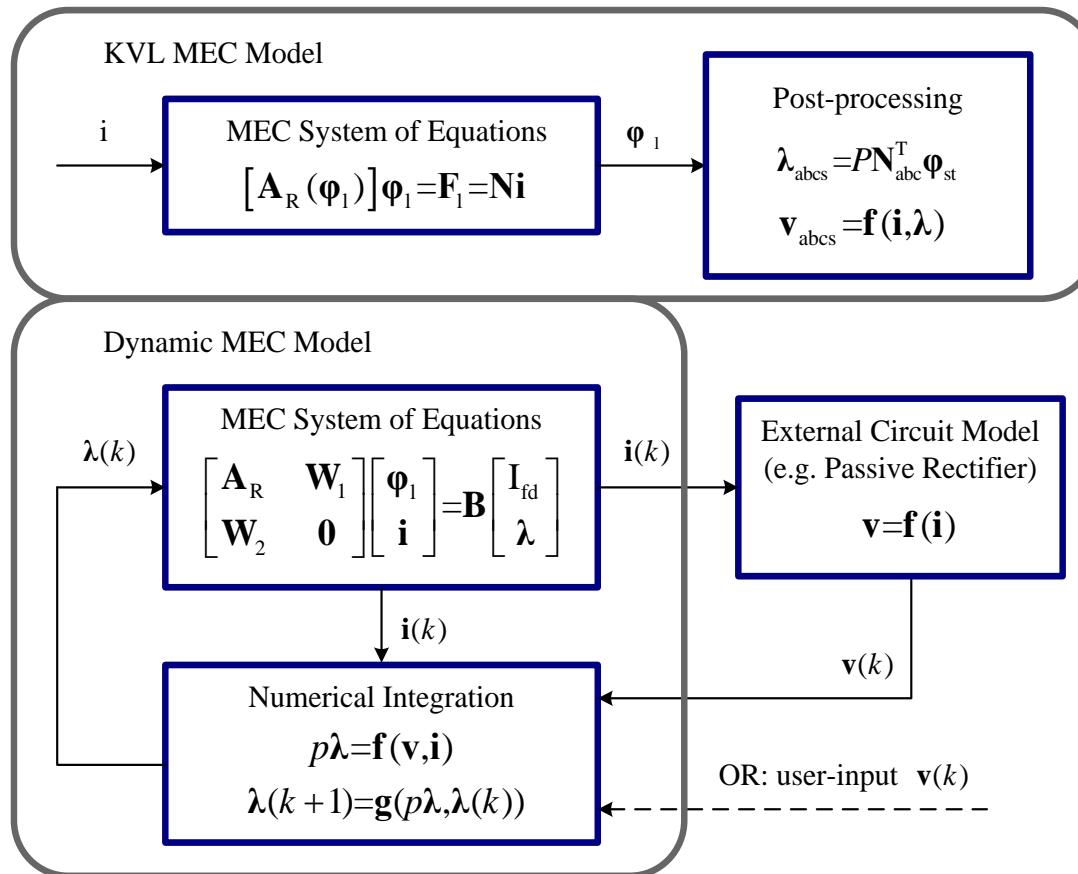
where $\mathbf{N}_{dp}^{(np \times nd)}(j, k) = 1$, if the k^{th} damper winding current is in the j^{th} rotor pole loop.

$\mathbf{N}_{dp}^{(np \times nd)}(j, k) = 0$, otherwise.

In this case,

$$\mathbf{F}_{rp}^{(5 \times 1)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{i}_{dp}^{(3 \times 1)}$$

Dynamic System Structure



Restructuring the MEC Model

1) Expand the KVL MEC system: $\mathbf{A}_R \Phi_1 - \mathbf{N}_{l,abc} \mathbf{i}_{abcs} - \mathbf{N}_{l,dp} \mathbf{i}_{dp} = \mathbf{N}_{l,fd} \mathbf{I}_{fd}$

2) Relate the flux and flux linkage:

$$\lambda_{abcs} = P \mathbf{N}_{l,abc} \Phi_{st}, \quad \lambda_{dp}^{(nd \times 1)} = \mathbf{M}_{l,dp}^{(nd \times nl)} \Phi_1^{(nl \times 1)} = \begin{bmatrix} \mathbf{0}^{(nd \times (nl-n))} & \mathbf{M}_{l,dp_sub}^{(nd \times nd)} \end{bmatrix} \Phi_1^{(nl \times 1)}$$

In this case,

$$\begin{bmatrix} \lambda_{dp,1} \\ \lambda_{dp,2} \\ \lambda_{dp,3} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & 1 \end{bmatrix}}_{\mathbf{M}_{l,dp_sub}} \begin{bmatrix} \phi_{rp1} \\ \phi_{rp3} \\ \phi_{rp5} \end{bmatrix}$$

3) Dynamic MEC system of equations:

$$\begin{bmatrix} \mathbf{A}_R & -\mathbf{N}_{l,abc} & -\mathbf{N}_{l,dp} \\ \mathbf{N}_{l,abc}^T & 0 & 0 \\ \mathbf{M}_{l,dp} & 0 & 0 \end{bmatrix} \begin{bmatrix} \Phi_1 \\ \mathbf{i}_{abcs} \\ \mathbf{i}_{dp} \end{bmatrix} = \begin{bmatrix} \mathbf{N}_{l,fd} & 0 & 0 \\ 0 & \mathbf{I}/P & 0 \\ 0 & 0 & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{I}_{fd} \\ \lambda_{abcs} \\ \lambda_{dp} \end{bmatrix}$$

Dynamic System with Scaling

Apply qd transformation:

$$\underbrace{\begin{bmatrix} \mathbf{A}_R & -f_{scale}\mathbf{N}_{l,abc}(\mathbf{K}_s)^{-1} & -f_{scale}\mathbf{N}_{l,dp} \\ f_{scale}\mathbf{K}_s\mathbf{N}_{l,abc}^T & 0 & \\ f_{scale}\mathbf{M}_{l,dp} & & \end{bmatrix}}_{\mathbf{A}_{dyn}} \begin{bmatrix} \boldsymbol{\Phi}_1 \\ \mathbf{i}_{qd0s,scl} \\ \mathbf{i}_{dp,scl} \end{bmatrix} = \begin{bmatrix} \mathbf{N}_{l,fd} & 0 & 0 \\ 0 & f_{scale}\mathbf{I}/P & 0 \\ 0 & 0 & f_{scale}\mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{I}_{fd} \\ \boldsymbol{\lambda}_{qd0s} \\ \boldsymbol{\lambda}_{dp} \end{bmatrix}$$

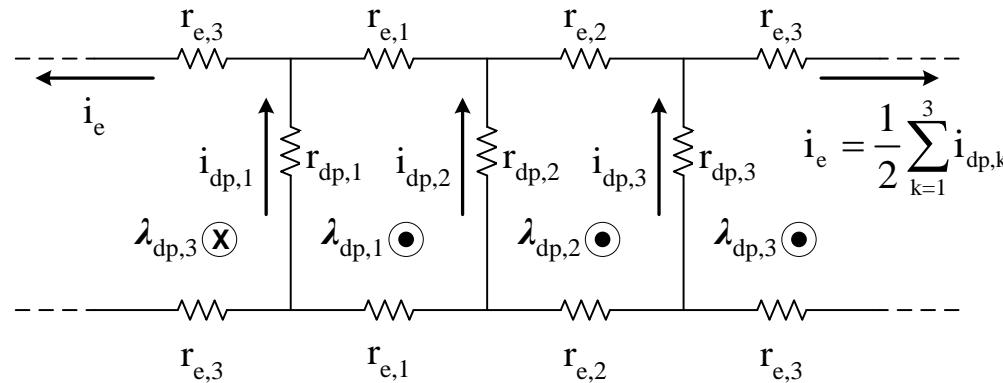
where $\mathbf{i}_{qd0s} = f_{scale}\mathbf{i}_{qd0s,scl}$, $\mathbf{i}_{dp} = f_{scale}\mathbf{i}_{dp,scl}$, $f_{scale} = 10^3$

Compare to the structure block diagram,

$$\mathbf{W}_1 = \begin{bmatrix} -f_{scale}\mathbf{N}_{l,abc}(\mathbf{K}_s)^{-1} & -f_{scale}\mathbf{N}_{l,dp} \end{bmatrix} \quad \mathbf{W}_2 = \begin{bmatrix} f_{scale}\mathbf{K}_s\mathbf{N}_{l,abc}^T \\ f_{scale}\mathbf{M}_{l,dp} \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} \mathbf{N}_{l,fd} & 0 & 0 \\ 0 & f_{scale}\mathbf{I}/P & 0 \\ 0 & 0 & f_{scale}\mathbf{I} \end{bmatrix}$$

State Equations of Damper Bars



From Ohm's and Faraday's laws,

$$p\lambda_{dp} = \mathbf{T}_{dp} \mathbf{i}_{dp}$$

where

$$\mathbf{T}_{dp} = \begin{bmatrix} r_{dp,1} + r_{e,1} & -r_{dp,2} - r_{e,1} & -r_{e,1} \\ r_{e,2} & r_{dp,2} + r_{e,2} & -r_{dp,3} - r_{e,2} \\ r_{dp,1} + r_{e,3} & r_{e,3} & r_{dp,3} + r_{e,3} \end{bmatrix}$$

State Equations of Stator Windings

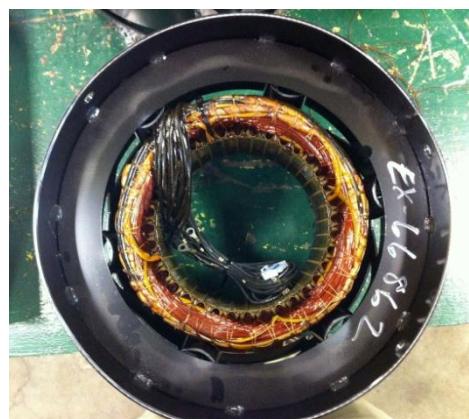
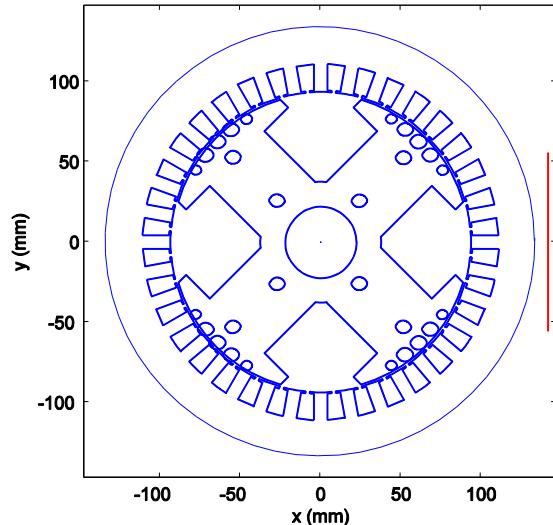
Rearrange the voltage equation:

$$p\lambda_{qd0s} = \mathbf{v}_{qd0s} - r_s \mathbf{i}_{qd0s} - \omega \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \lambda_{qd0s}$$

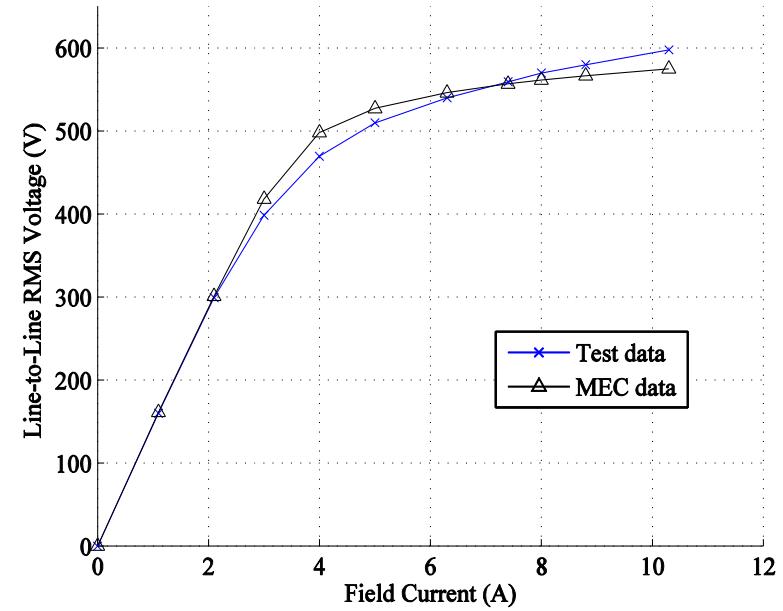
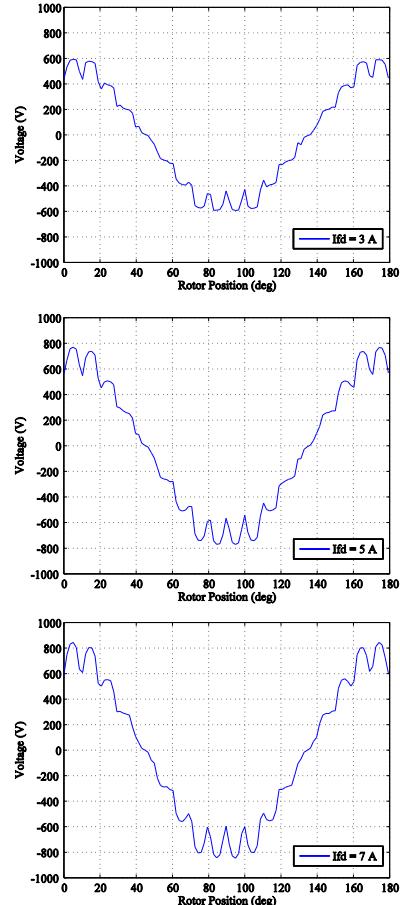
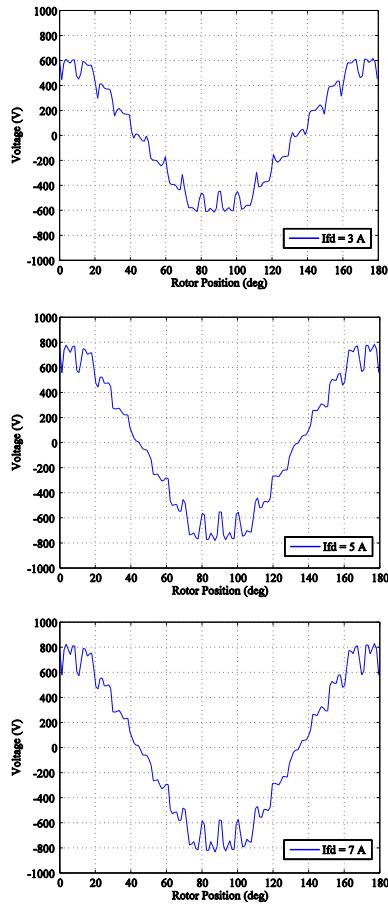
Calculation of resistive loss:

$$P_{res} = \underbrace{\frac{3}{2\pi} \int_0^{2\pi} r_s i_{as}^2(\theta_r) d\theta_r + r_{fd} i_{fd}^2}_{stator+field} + \underbrace{\frac{P}{2\pi} \sum_{k=1}^{nd} \int_0^{2\pi} [r_{dp,k} i_{dp,k}^2(\theta_r) + r_{e,k} i_{e,k}^2(\theta_r)] d\theta_r}_{damper}$$

Hardware Validation



Open Circuit Voltage



Field currents: 0-10.2 A
Rotor speed: 1800 rpm
Maximum error: 5%

Balanced 3-Phase Load Test

Conditions:

- Rotor speed: 1800 rpm
- RMS line-line voltage: 480 V
- Power factor: 0.8 lagging
(parallel RL load)
- $P_{mech} = 303 \text{ W}$
- $r_{brush} = 1 \Omega$
- Loss of exciter is not modeled

Load	RMS values of Phase current (A)		Average input torque (Nm)		Output power (kW)	
	MEC	Test	MEC	Test	MEC	Test
1	4.8	4.5	21.01	19.98	3.1775	2.9858
2	8.0	7.6	33.81	32.26	5.3641	5.0707
3	12.0	11.4	49.95	47.78	7.9783	7.5915
4	15.9	15.2	66.11	64.16	10.4445	10.1030

Average input torque:

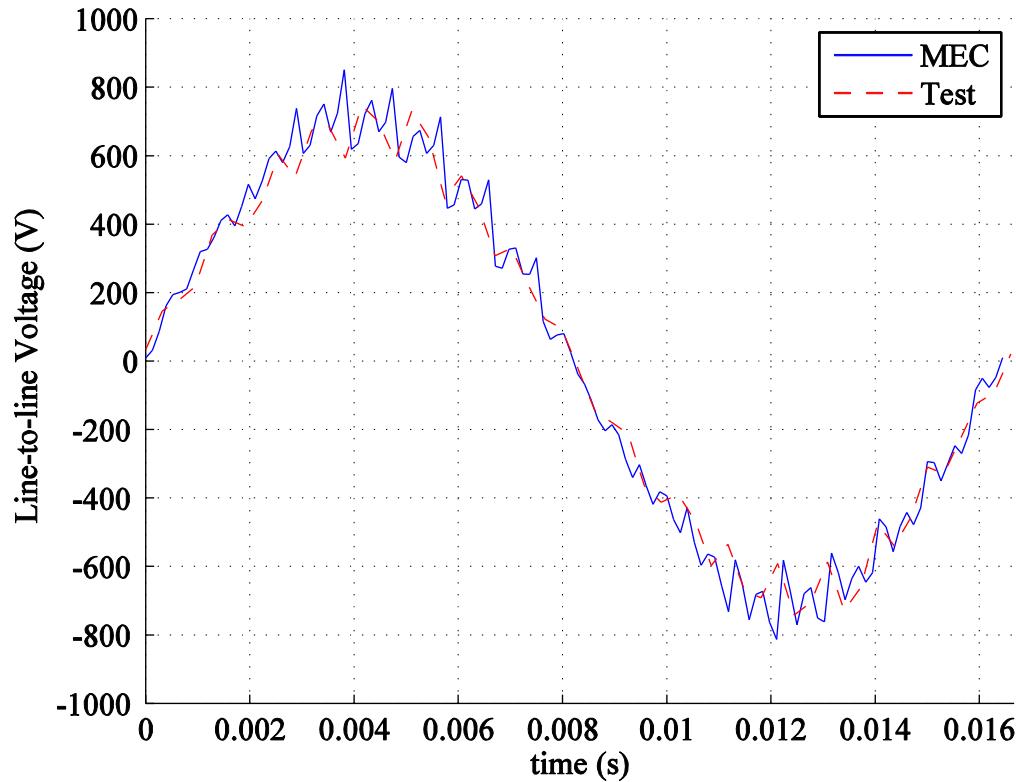
$$T_{in_avg} = \frac{T_e \omega_{rm} + P_{mech} + P_{core}}{\omega_{rm}}$$

Output power:

$$P_{out} = T_e \omega_{rm} - P_{res}$$

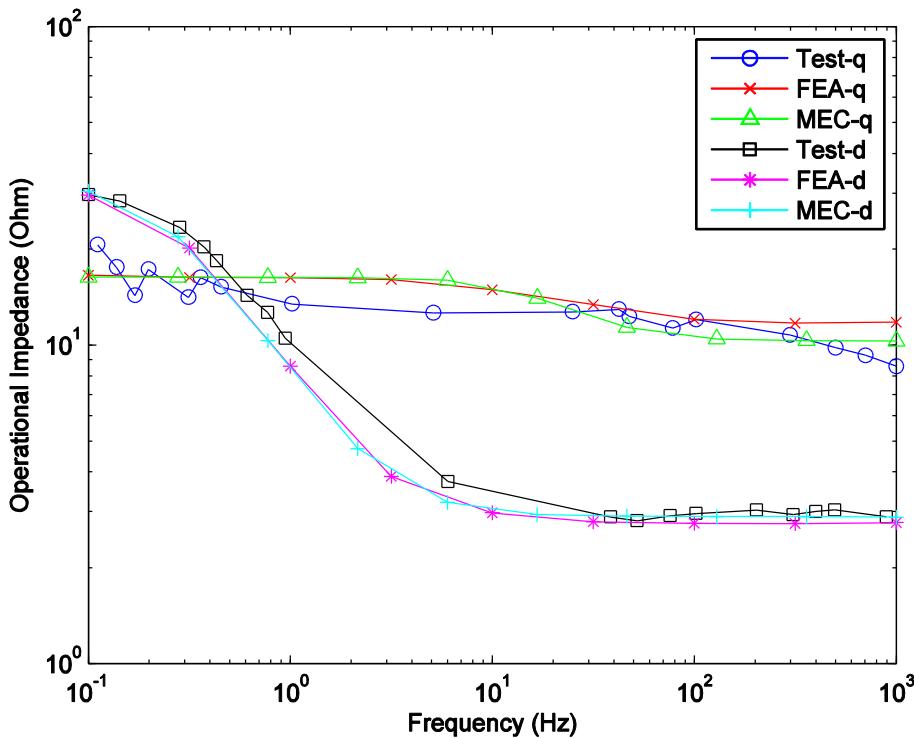
Load	MEC				Test	
	P_{s+f} (W)	P_{core} (W)	P_{dp} (W)	$P_{core+dp}$ (W)	P_{s+f} (W)	$P_{core+dp}$ (W)
1	235.5	232.3	12.5	244.7	229.5	247.9
2	431.2	241.8	32.9	274.7	414.5	292.8
3	793.2	254.2	86.4	340.6	757.6	354.4
4	1267.4	265.3	181.9	447.2	1211.2	477.0

Voltage Waveforms



The error of RMS values is approximately 5%.

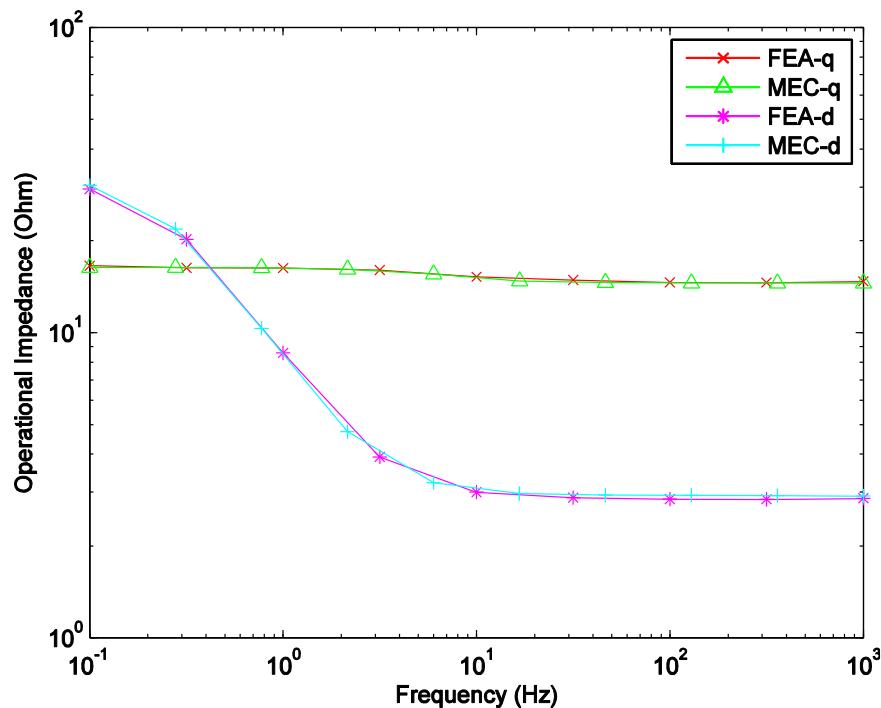
Standstill Frequency Response



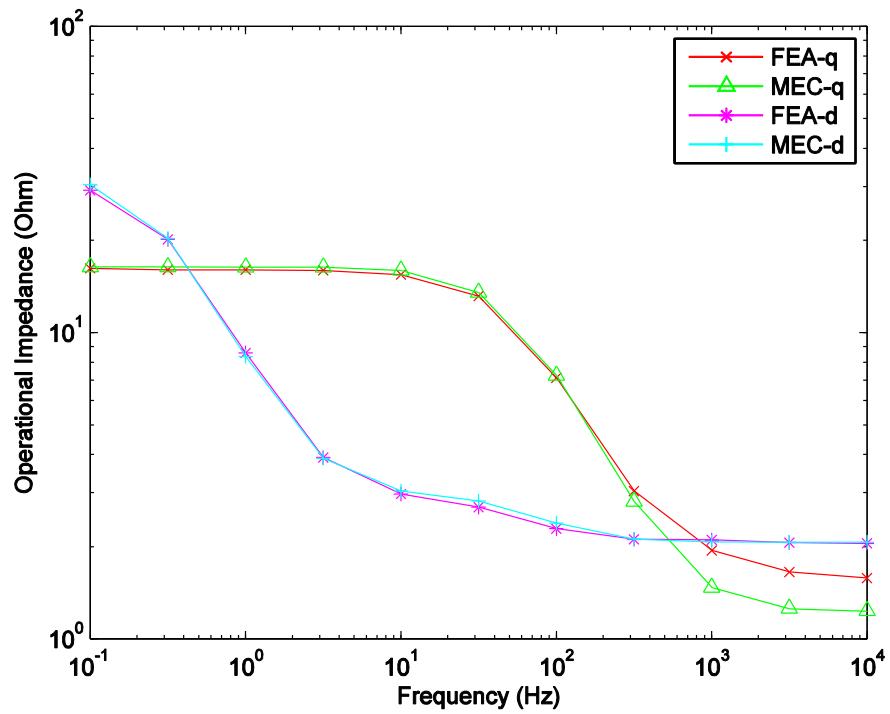
Low-frequency asymptote: magnetizing impedances.
High-frequency asymptote: subtransient impedances.

- At low frequencies (< 0.4 Hz)
Match closely.
- At mid frequencies (> 0.4 Hz, <20 Hz)
Additional conduction path that exists between the copper plates and the rotor shaft.
- At higher frequencies (> 100 Hz)
Error between MEC and FEA comes from the modeling of rotor pole tip leakage flux path.
Lower test-*q* due to eddy currents.

Different Depth of Damper Bars

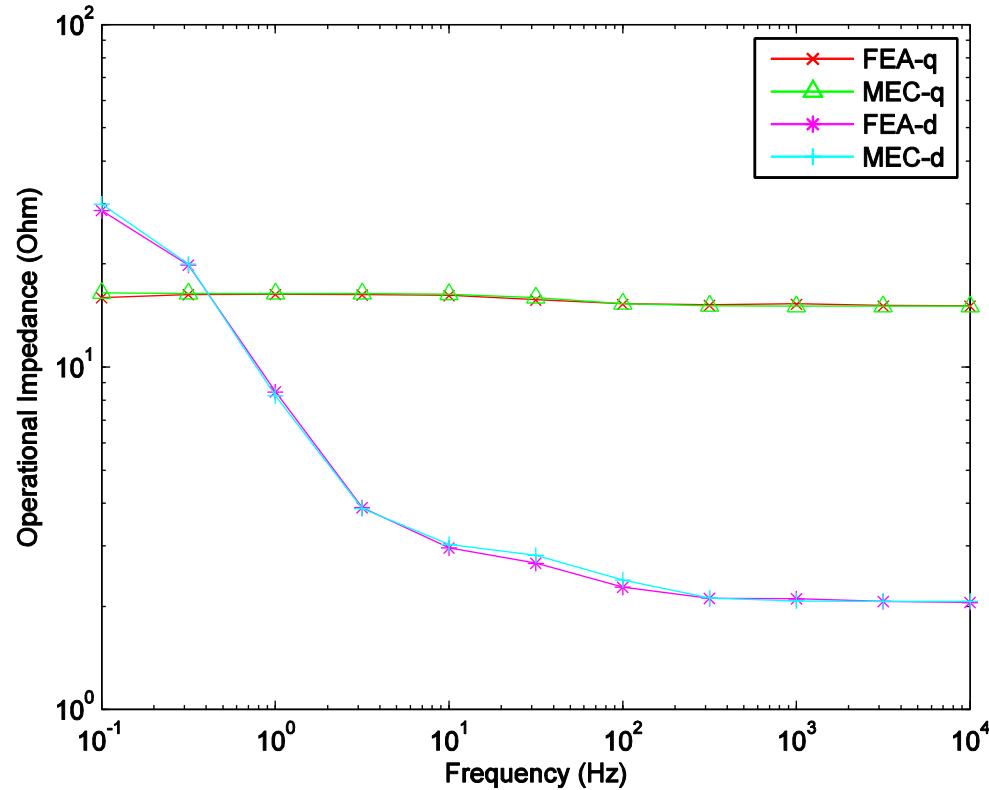


$$\alpha_{dp} = 0.5$$

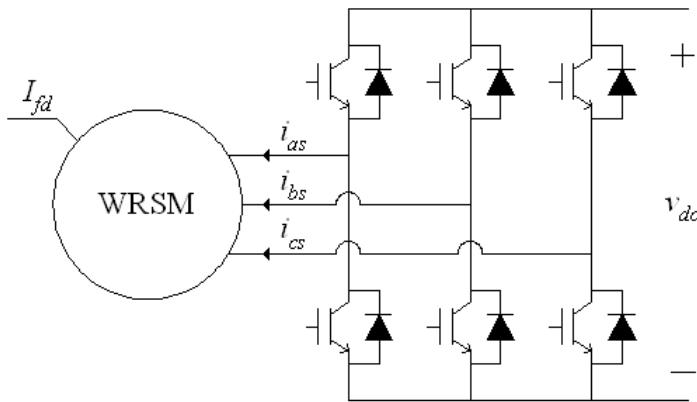


$$\alpha_{dp} = 0.0001$$

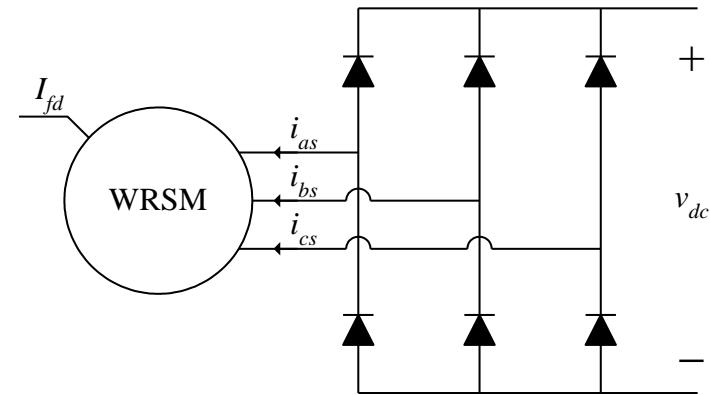
Influence of the Leakage Path Between Poles



25 MW WRSM/Rectifier Design



WRSM/active rectifier

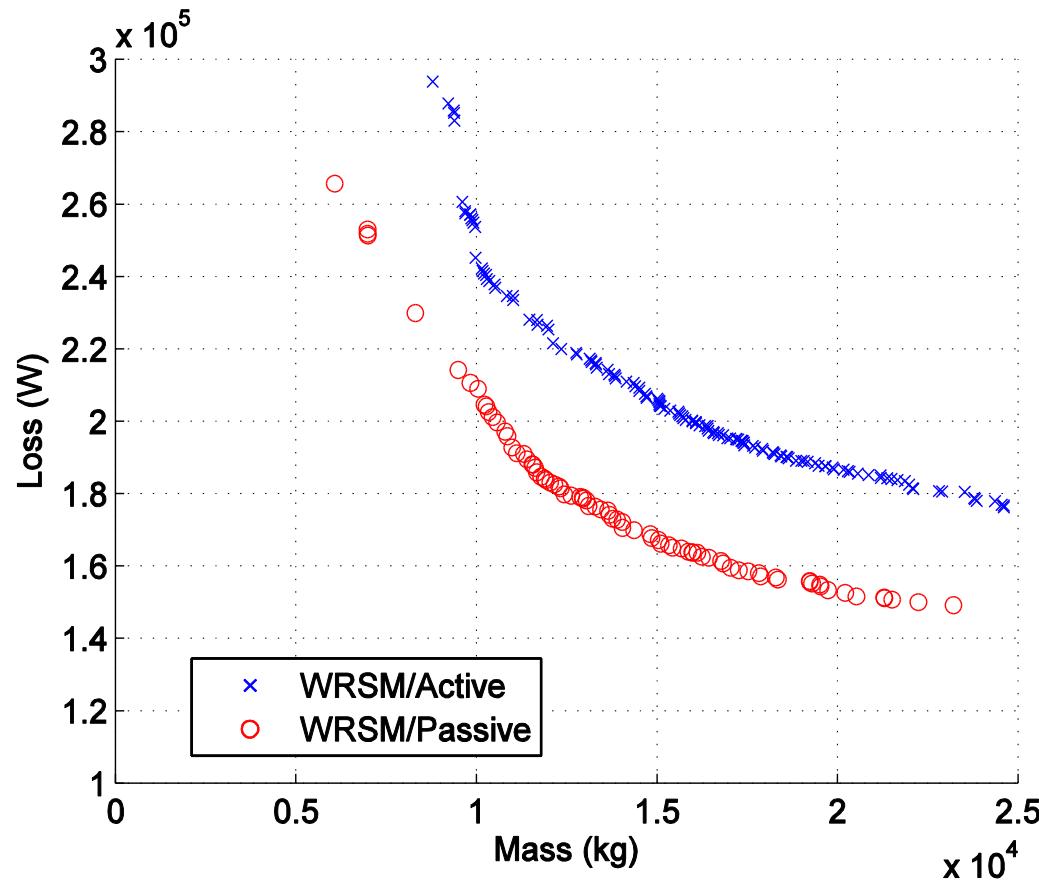


WRSM/pассив rectifier

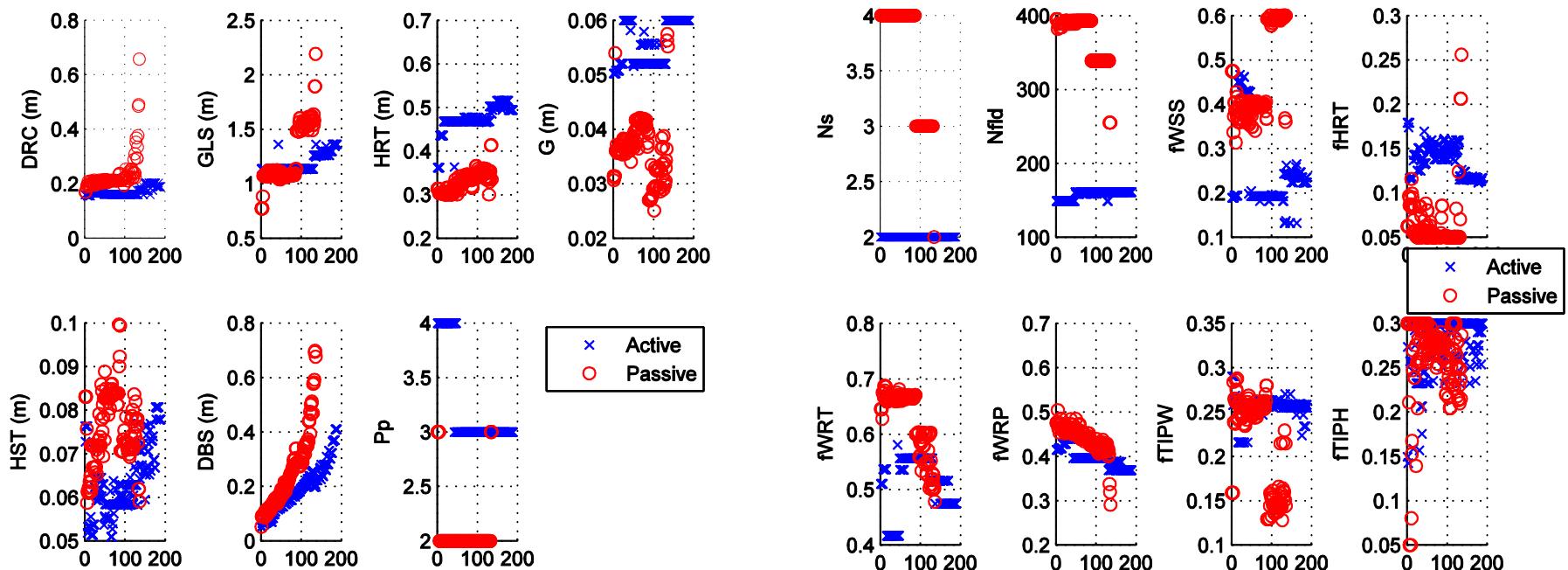
$$\boldsymbol{\theta} = [d_{rc} \quad l \quad d_{rt} \quad g \quad d_{st} \quad d_{bs} \quad fw_{ss} \quad fh_{rt} \quad fw_{rt} \quad \dots \\ fw_{rp} \quad N_s \quad I_s \quad \beta \quad N_{fd} \quad I_{fd} \quad P_p \quad ftipw \quad ftiph]^T$$

$$\boldsymbol{\theta} = [d_{rc} \quad l \quad d_{rt} \quad g \quad d_{st} \quad d_{bs} \quad fw_{ss} \quad fh_{rt} \quad fw_{rt} \quad \dots \\ fw_{rp} \quad N_s \quad N_{fd} \quad I_{fd} \quad P_p \quad ftipw \quad ftiph \quad r_{dt} \quad d_{num} \quad d_{con}]^T$$

Pareto Front

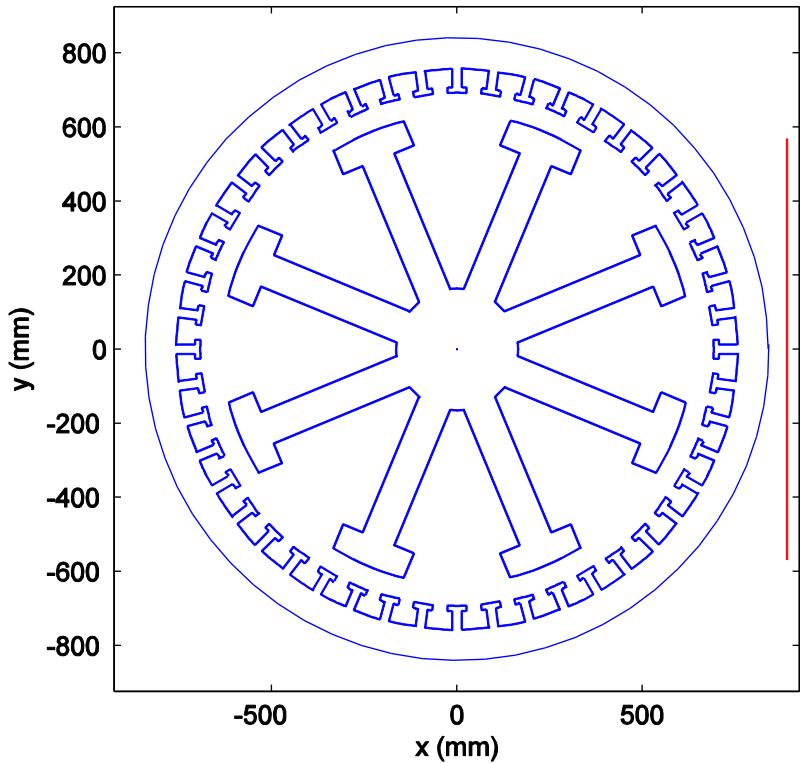


Genes Distribution

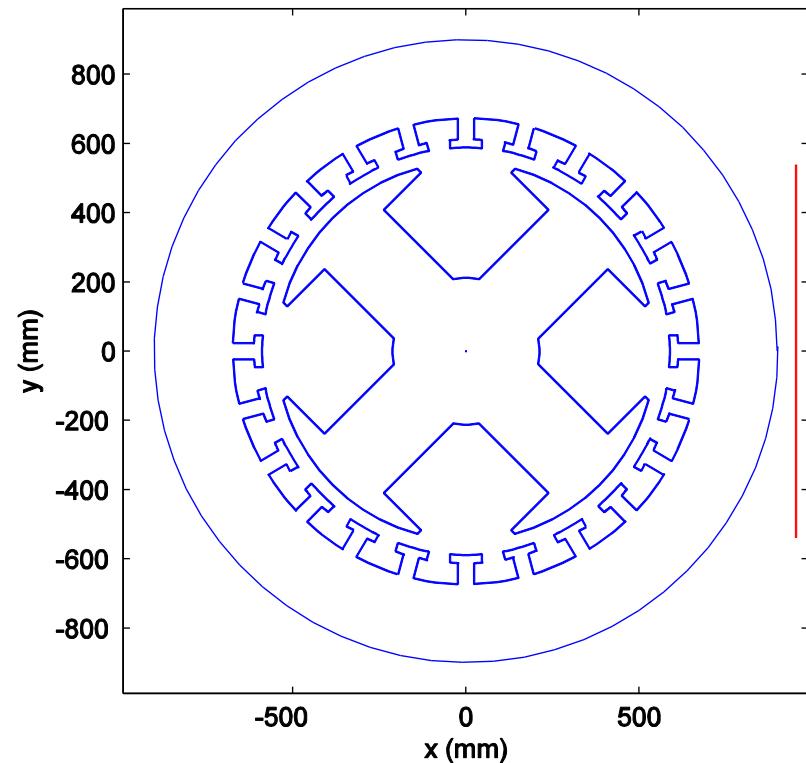


WRSM/active rectifier: larger height of rotor teeth (HRT), airgap length (G), and pole pair (Pp).
 WRSM/passive rectifier: larger stack length (GLS), stator turns (Ns) and field turns (Nfld).

Example Machines



WRSM/active rectifier



WRSM/пассив rectifier

Conclusions

- Developed a voltage-input dynamic MEC model that includes damper bar currents dynamics.
 - Enables exploration of alternative damper configurations
 - Enables multi-objective design of machine/pассив rectifiers
 - Readily extended to multi-phase machines
- Initial study utilized to compare Passive/Active designs
 - Surprising result is that the machine for the passive designs are less massive for a given specified loss