Evolutionary Design of PMSM Machines

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Grainger Seminar Series

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  – Mark Snyder (Purdue Undergraduate)
Manual Design Approach

Analysis

Design Equations

Numerical Analysis

Design Revisions

Final Design
Optimization Based Design and Analysis (Evolutionary or Otherwise)

• Pose Design Problem As An Optimization Problem
  – Can be single or multi-objective
  – Systematically encode design constraints and objectives into objective (fitness) function

• Reality Check: Problem Properties Not Always Friendly
  – Not differentiable
  – Not convex
  – Many local extrema
  – No unique global optimum
Optimization Methods

• Classic Methods
  – Gradient Methods
  – Newton’s Method
  – Conjugate Direction Methods
  – Quasi-Newton Methods
  – Nedler-Mead Simplex Method

• Populations Based Methods
  – Population Based Classical Methods
  – Monte-Carlo
  – Genetic Algorithms
  – Particle Swarm Algorithms
Evolutionary Design Approach

Evolutionary Environment

Detailed Analysis

Fitness Function
Modern (Real Coded) GA Execution

START

Initialization

Diversity control

Mating & Crossover & Death

Selection

Mutation

Migration

Scaling

Fitness evaluation

Elitism

Random search

Report Plot

STOP?

END
Facts of Life in Design Optimization

• On finding the global optimum ....

• On design repeatability ....
Analysis Options

- The hard part is predicting the magnetics (the incremental inductance).
- Options include
  - Analytical methods
  - Magnetic equivalent circuits
  - High fidelity magnetic equivalent circuits
  - 2-D Finite element analysis
  - 3-D Finite element analysis
Example: Design of a Permanent Magnet Synchronous Machine

- **Specifications**
  - 10 Nm at 2000 RPM with an RMS current of 10 A
  - The machine should be as torque dense as possible subject to a loss of 94 W


## Parameter Encoding

### Direct Method

\[
\theta = [m_r \quad m_s \quad m_p \quad \phi_i \quad \alpha_{pm} \quad r_r \quad d_m \quad g \quad \alpha_s \quad d_b \quad l \quad N_p^* \quad \alpha_3^* \quad m_c \quad \alpha_c \quad \alpha_f]^T
\]

### Indirect Method

\[
\theta = [m_r \quad m_s \quad m_p \quad \phi_i \quad \alpha_{pm} \quad \alpha_r \quad \alpha_{d_m} \quad \alpha_g \quad \alpha_{d_s} \quad \alpha_{d_h} \quad l \quad \alpha_{N_p} \quad \alpha_{\alpha_3} \quad m_c \quad \alpha_{\alpha_c} \quad \alpha_f]^T
\]
## Constraints

### Table IV
**Design Constraints**

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_1(\theta) = \ln(B_{t,max}, B_{si,max}^*) )</td>
<td>Tooth flux density</td>
</tr>
<tr>
<td>( c_2(\theta) = \ln(B_{b,max}, B_{si,max}^*) )</td>
<td>Backiron flux density</td>
</tr>
<tr>
<td>( c_3(\theta) = \ln(B_{ib,max}, B_{si,max}^*) )</td>
<td>Tooth base flux density</td>
</tr>
<tr>
<td>( c_4(\theta) = \ln(B_{ri,max}, B_{ri,max}^*) )</td>
<td>Rotor iron flux density</td>
</tr>
<tr>
<td>( c_5(\theta) = \det(H_{m,min}, H_{m,min}^*) )</td>
<td>Demagnetization</td>
</tr>
<tr>
<td>( c_6(\theta) = \ln(I_s / a_c, J_{c, max}^*) )</td>
<td>Current density constraint</td>
</tr>
<tr>
<td>( c_7(\theta) = \ln(\alpha_{sf,max}, \alpha_{sf,max}^*) )</td>
<td>Slot fill constraint</td>
</tr>
<tr>
<td>( c_8(\theta) = \det(g, k_{gap} r_r) )</td>
<td>Air gap constraint</td>
</tr>
<tr>
<td>( c_9(\theta) = \det(T_e, T_e^*) )</td>
<td>Torque constraint</td>
</tr>
<tr>
<td>( c_{10}(\theta) = \ln(P_{loss}, P_{max}) )</td>
<td>Power loss constraint</td>
</tr>
</tbody>
</table>
Fitness Function Construction

• Our Objective

\[ \hat{f}(\theta) = \frac{T_e^*}{\pi (r_r + d_m + g + d_b)^2 l} \]

• Continuous Fitness Function

\[ f_1(\theta) = \hat{f}(\theta) c(\theta) \quad c(\theta) = \prod_{i=1}^{C} c_i(\theta) \]

• Discontinuous Fitness Function

\[ f_2(\theta) = \begin{cases} 
\left( \sum_{i=1}^{C} c_i \right) - C & c < 1 \\
\hat{f}(\theta) & c \geq 1
\end{cases} \quad c = \min(c_1, c_2, ..., c_C) \]
Machine Control

- Machine connected to a current controlled inverter

\[ i_{as} = \sqrt{2} I_s \cos(\theta_r + \phi_i) \]

\[ i_{bs} = \sqrt{2} I_s \cos(\theta_r + \phi_i - 2\pi / 3) \]

\[ i_{cs} = \sqrt{2} I_s \cos(\theta_r + \phi_i + 2\pi / 3) \]

- Conductor distribution: sinusoidal

\[ n_{as}(\phi_{sm}) = N_p \left( \sin \left( \frac{P}{2} \phi_{sm} \right) - \alpha_3 \sin \left( \frac{3}{2} \phi_{sm} \right) \right) \]

\[ n_{bs}(\phi_{sm}) = N_p \left( \sin \left( \frac{P}{2} \phi_{sm} - \frac{2\pi}{3} \right) - \alpha_3 \sin \left( \frac{3}{2} \phi_{sm} \right) \right) \]

\[ n_{cs}(\phi_{sm}) = N_p \left( \sin \left( \frac{P}{2} \phi_{sm} + \frac{2\pi}{3} \right) - \alpha_3 \sin \left( \frac{3}{2} \phi_{sm} \right) \right) \]
Flux Density in Permanent Magnet
Flux Density in Air Gap

\[
B(r, \phi_{sm}) = \frac{r}{r_r} \frac{F_m(\phi_{sm}) - i_{enc}(\phi_{sm})}{R_p(\phi_{sm}) + R_g}, \quad r_r \leq r \leq r_s
\]

\[
i_{enc}(\phi_{sm}) = -\frac{3\sqrt{2}N_p I_s}{P} \cos \left( \frac{P}{2} \phi_{sm} - \theta_r - \phi_i \right)
\]

\[
R_p(\phi_{sm}) = \frac{r_r}{\mu_0 \mu_{rm} (\phi_{sm} - \theta_{rm})} \ln \left( 1 + \frac{d_m}{r_r} \right)
\]

\[
F_m(\phi_{sm}) = \frac{d_m}{\mu_0 \mu_{rm} (\phi_{sm} - \theta_{rm})} B_m(\phi_{sm} - \theta_{rm})
\]

\[
R_g = \frac{r_r}{\mu_0} \ln \left( 1 + \frac{g}{r_r + d_m} \right)
\]
Key Lumped Parameters

\[ L_{qm} = \frac{6l_{eff}r_N^2}{P^2} \left[ \frac{\pi (1 - \alpha_{pm}) + \sin(\pi \alpha_{pm})}{R_{sp} + R_g} + \frac{\pi \alpha_{pm} - \sin(\pi \alpha_{pm})}{R_{pm} + R_g} \right] \]

\[ L_{dm} = \frac{6l_{eff}r_N^2}{P^2} \left[ \frac{\pi (1 - \alpha_{pm}) - \sin(\pi \alpha_{pm})}{R_{sp} + R_g} + \frac{\pi \alpha_{pm} + \sin(\pi \alpha_{pm})}{R_{pm} + R_g} \right] \]

\[ \lambda_m = \frac{8r_N^2}{P(R_{pm} + R_g)} \frac{d_m}{\mu_0 \mu_{rm}} B_{pm} \sin \left( \frac{\pi \alpha_{pm}}{2} \right) \]
End Effects on Magnetizing Inductance
Axially Compensated Length

\[ P_1 = \frac{\mu_0 w_1}{\pi} \ln \left(1 + \frac{\pi d_1}{g_1}\right) \]

\[ P_2 = \frac{\mu_0 w_2}{\pi} \ln \left(1 + \frac{\pi d_2}{g_2}\right) \]

\[ P_{\text{end}} = 2(P_1 + P_2 + P_3 + P_4) \]

\[ P_3 = \frac{\mu_0}{\pi^2} \left[ 2\pi d_3 \ln \left(1 + \frac{\pi v_3}{2g_3 + 2\pi d_3}\right) + (\pi v_3 + 2g_3) \ln \left(1 + \frac{2\pi d_3}{\pi v_3 + 2g_3}\right) - 2g_3 \ln \left(1 + \frac{\pi d_3}{g_3}\right) \right] \]

\[ P_4 = \frac{\mu_0}{\pi^2} \left[ 2\pi d_4 \ln \left(1 + \frac{\pi v_4}{2g_4 + 2\pi d_4}\right) + (\pi v_4 + 2g_4) \ln \left(1 + \frac{2\pi d_4}{\pi v_4 + 2g_4}\right) - 2g_4 \ln \left(1 + \frac{\pi d_4}{g_4}\right) \right] \]
Slot Leakage Permeance
End Leakage Permeance

\[ l_r = \max(l_{ew}, d_{ew}) \]
\[ d_r = \min(l_{ew}, d_{ew}) \]

\[ P_{el,1} = \frac{\mu_0 L_{seg}}{d_r^2 l_r^2} \left[ \frac{d_r^4}{32} + \frac{d_r^3}{16} (l_r - d_r) + \frac{d_r^2}{64} (l_r - d_r)^2 \right. \]
\[ \left. - \frac{d_r}{64} (l_r - d_r)^3 + \frac{1}{128} (l_r - d_r)^4 \ln \left( 1 + \frac{2d_r}{l_r - d_r} \right) \right] \]

\[ R = \frac{(l_{eo} + l_{ew} + x)}{d_b L_{seg} \mu_0} + \frac{\left( d_w + \frac{d_b}{2} + y \right)}{2 x L_{seg} \mu_0} + \frac{(l_{eo} + l_{ew} + x)}{2 y L_{seg} \mu_0} \]

\[ P_{el,2} = \frac{1}{R_{\text{min}}} \]
Hardware Validation
## Hardware Validation

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_s$</td>
<td>2.22 Ω</td>
<td>2.22 Ω</td>
<td>2.22 Ω</td>
<td>2.22 Ω</td>
<td>2.22 Ω</td>
<td>2.29 Ω</td>
<td>3.1%</td>
</tr>
<tr>
<td>$\lambda_m$</td>
<td>.295 Vs</td>
<td>.295 Vs</td>
<td>.295 Vs</td>
<td>.293 Vs</td>
<td>.295 Vs</td>
<td>.308 Vs</td>
<td>4.2%</td>
</tr>
<tr>
<td>$L_q$</td>
<td>16.45 mH</td>
<td>15.88 mH</td>
<td>15.69 mH</td>
<td>16.20 mH</td>
<td>16.06 mH</td>
<td>15.19 mH</td>
<td>5.4%</td>
</tr>
<tr>
<td>$L_d$</td>
<td>14.38 mH</td>
<td>14.87 mH</td>
<td>15.19 mH</td>
<td>14.48 mH</td>
<td>14.73 mH</td>
<td>15.26 mH</td>
<td>3.6%</td>
</tr>
</tbody>
</table>
Sample Design

(Direct Encoding; Discontinuous Fitness)
Sample Design

Table IV
Design Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotor Material</td>
<td>M47</td>
</tr>
<tr>
<td>Type</td>
<td>M19</td>
</tr>
<tr>
<td>Stator Material</td>
<td>NdFeB-30</td>
</tr>
<tr>
<td>Type</td>
<td>Backiron (cm)</td>
</tr>
<tr>
<td>Permanent</td>
<td>Active Length</td>
</tr>
<tr>
<td>Magnet Type</td>
<td>(cm)</td>
</tr>
<tr>
<td>Current Phase (Degrees)</td>
<td>Fundamental (cm)</td>
</tr>
<tr>
<td>Advance Density (Percent)</td>
<td>Conductor Density</td>
</tr>
<tr>
<td>Permanent</td>
<td>3rd Harmonic</td>
</tr>
<tr>
<td>Magnet Fraction (Percent)</td>
<td>Turn Density</td>
</tr>
<tr>
<td>Slot Fraction (Percent)</td>
<td>Conductor Type</td>
</tr>
<tr>
<td>Rotor Iron Radius (cm)</td>
<td>Conductor Area (mm²)</td>
</tr>
<tr>
<td>Permanent Magnet Depth (cm)</td>
<td>Fillet Fraction</td>
</tr>
<tr>
<td>Air Gap (mm)</td>
<td></td>
</tr>
</tbody>
</table>

Tooth Flux Density, T

Backiron Flux Density, T

θ, Degrees
Algorithm Comparisons

One Region / Neighborhood

**GA**
Population Size: 2000
Generations: 2000
Evaluations: 2,400,000

**PSO**
Swarm Size: 1549
Iterations: 1549
Evaluations: 2,400,000
Algorithm Comparisons
Thirty Regions / Neighborhoods

GA
Population Size: 2000
Generations: 2000
Evaluations: 2,400,000

PSO
Swarm Size: 1549
Iterations: 1549
Evaluations: 2,400,000
Design Uniqueness
(GA, Direct Enc., Disc. Fitness)
Design a Soldier Portable Generator

• Specifications
  – 400 V dc Bus
  – 3 kW Electrical Output Power
Candidate Designs

Path of Integral for Section 7.2.1

Stator Iron

PM: S-pole
PM: N-pole

Rotor Iron

PM: S-pole
PM: N-pole

Stator Iron

PM: S-pole
PM: N-pole

Integration Path

Stator Slot

Shaft

Spacer

Rotor Core

Stator

Rotor

Core

Shaft
Machine Comparison

\[ \bar{c} = \frac{1}{N} \sum_{n=1}^{N} c_n \]

\[ f = \begin{cases} 
\frac{1}{\varepsilon(c - 1)} \begin{bmatrix} 1 \\ 1 \end{bmatrix} & \bar{c} < 1 \\
\frac{1}{m} \begin{bmatrix} 1 \\ \bar{p}_{loss} \end{bmatrix} & \bar{c} \geq 1 
\end{cases} \]
Selecting a Specific Design

• Approach 1: Pick one
  – Advantage: easy
  – Disadvantage: may not be on the front

• Approach 2: Run a single objective optimization with added constraint on loss
Single Objective Optimization

- For 2 kW design

\[ \bar{c} = \frac{1}{N} \sum_{n=1}^{N} c_n \]

\[ f = \begin{cases} 
\varepsilon (\bar{c} - 1) & \bar{c} < 1 \\
\frac{1}{m} e \left( \frac{P_{\text{req}} - P_{\text{out}}}{P_{\text{req}}} \right) & \bar{c} \geq 1 
\end{cases} \]

- Added constraint of loss of 257 W
- 90% efficiency for generator / rectifier
Design Evolution
Design Report

- Mass: (kg) 7.9715
  - Rotator Steel Mass (less shaft) (kg): 1.2523
  - Stator Iron Mass (kg): 4.8969
  - Stator Copper Mass (kg): 1.4794
  - Magnet Mass (kg): 0.34289
  - Rotor Material Type: M19
  - Stator Material Type: M19
  - Permanent Magnet Type: NdFeB-30
  - RMS Line Current (A): 13.5153
  - Current Phase Advance (Degrees): 171.332
  - Permanent Magnet Fraction (Percent): 36.5488
  - Slot Fraction (Percent): 24.0531
  - Rotor Iron Radius (cm): 3.1633
  - Permanent Magnet Depth (cm): 0.73316
  - Air Gap (mm): 1.0185
  - Slot Depth (cm): 2.1585
  - Depth of Backiron (cm): 0.96692
  - Active Length (cm): 7.8083
  - Minimum Total Length (cm): 12.4086
  - Outside Diameter (cm): 14.2473
  - Number of Slots: 24
  - Fundamental Turns Density (turns/rad): 40.9794
  - 3rd Harmonic Turns Density (percent): 21.97
  - Conductor Type: Copper
  - Conductor Area (mm²): 2.0816
  - Slot Opening: 2.5173 mm
  - American Wire Gauge: 14

- Stator Tooth Flux Density / Saturation Point: 0.99082
- Stator Tooth Base Flux Density / Saturation Point: 0.95629
- Stator Backiron Flux Density / Saturation Point: 0.98946
- Permanent Magnet Demagnetization: 0.493
- Current Density / Allowable Current Density: 0.85207
- Maximum Slot Fill / Allowed Slot Fill: 0.55345
- Air Gap / Minimum Allowed Air Gap: 3.2199
- RMS L-N Voltage / Maximum Allowed Voltage: 0.83949
- Power Factor = 0.99276
- Torque: -6.6951
- Corrected Torque: -6.7866
- Conduction Losses: 120.8122
- Core Losses in Teeth: 14.1565
- Core Losses in Backiron: 20.0615
- Total Machine Loss: 155.0302
- Rectifier Loss: 73.0084
- Rectifier Efficiency: 0.96935
- Machine Efficiency: 0.9389

- Stator Resistance (Ohms) = 0.22046
- Q-Axis Inductance (mH) = 1.6345
- D-Axis Inductance (mH) = 1.6345

- Flux Linkage Due to PM (Vs) = 0.11811
  - Q-Axis Voltage (V) = 80.624
  - D-Axis Voltage (V) = 22.4575
  - RMS L-N Voltage (V) = 59.1801
  - Q-Axis Current (A) = -18.8952
  - D-Axis Current (A) = -2.8806
  - RMS Current (A) = 13.5153
  - Electrical Power Out (W) = 2382.153
Packaged Hardware
System Waveforms

Purple: Output voltage (500 V/V)
Blue: DC-link voltage (500 V/V)
Green: Output current (200 A/V)
Yellow: Alt. current (200 A/V)

2.1 kW Output
System Waveforms

Yellow: a-b voltage (500 V/V)
Blue: b-c voltage (500 V/V)
Green: phase-a current (200 A/V)
Yellow: phase-b current (200 A/V)

2.1 kW Output
Performance Summary

- Alternator
  - Average Torque (estimated using phase currents/voltage) ~ 6.653 Nm
  - Input Average Power @3509 rpm = Te*speed ~ 2445 W
  - Output Power at Terminals = 2332 W
  - Efficiency ~ 95%

- Combined Active Rectifier/Inverter
  - Power Output = 2109 W
  - Power Input ~ 2332 W
  - Efficiency ~ 90%

- Overall Efficiency ~ 85%

- Mass
  - Alternator (Engine Mount) = 11.1 kg
  - Active Rectifier/Inverter (with enclosure) = 2.2 kg
  - Total = 13.3 kg

Baseline System
- Efficiency ~ 72%
- Mass = 17.8 kg

Savings Per Year:
- $15 Million
Model Fidelity

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Design Code</th>
<th>Modified Code</th>
<th>Constructed Machine</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flux Linkage due to PM (Vs)</td>
<td>0.118</td>
<td>0.118</td>
<td>0.115</td>
</tr>
<tr>
<td>Stator Winding Resistance (Ohm)</td>
<td>0.205</td>
<td>0.22</td>
<td>0.24</td>
</tr>
<tr>
<td>$q$-axis Stator Inductance (mH)</td>
<td>1.37</td>
<td>1.63</td>
<td>1.65</td>
</tr>
<tr>
<td>$d$-axis Stator Inductance (mH)</td>
<td>1.37</td>
<td>1.63</td>
<td>1.65</td>
</tr>
<tr>
<td>Magnetic Mass of Machine (kG)</td>
<td>7.99</td>
<td>7.97</td>
<td>8.5</td>
</tr>
<tr>
<td>Machine Efficiency (%)</td>
<td>94.2</td>
<td>93.8</td>
<td>95.3</td>
</tr>
</tbody>
</table>

Code modifications:
- End winding Offset (1.5 rather than 1 cm)
- Packing Factor Assumptions (0.7 rather than 0.4)

Rectifier/Inverter Efficiency Predicted in System Simulation (Neglecting Switching Losses) = 92%

Measured Rectifier/Inverter Efficiency = 90 %
Opinions

• Evolutionary design will become standard methodology for electromagnetic / electromechanical design
• Analytical and magnetic equivalent circuit methods are still important in the design process
• Use of multiple regions / neighborhoods is clearly beneficial
• Discontinuous fitness function with direct encoding seemed to be most effective
• Machines classes should be compared in terms of Pareto-Optimal fronts
• GA/PSO/Simulated Annealing/Bacteria Foraging/ etc. debate will go on forever.
GOSET

- **GOSET** stands for **Genetic Optimization System Engineering Tool**
- GOSET is a MATLAB based genetic algorithm toolbox for solving optimization problems
- GOSET Features
  - Wide range of choices for genetic operators
  - Single-objective optimization
  - Multi-objective optimization
  - Modular Structure
  - GUI Interface
  - GOSET DLL
- Available (at not cost) at:

Application Examples